

Modelling the Growth of Mean Top Height and Basal Area of *Eucalyptus grandis* in Zimbabwe

A report

submitted in partial fulfilment

of the requirements for the degree

of Master of Forestry Science

in the

University of Canterbury

by

Charles Chikono

University of Canterbury

1994

Table of Contents

| | |
|--|-----|
| List of Figures | i |
| List of Tables | iii |
| Abstract | 1 |
| Introduction | 1 |
| Forestry in Zimbabwe | 1 |
| The History of Eucalypts in Zimbabwe | 3 |
| Modelling in Zimbabwe | 6 |
| Objectives of the Study | 7 |
| Literature Review | 8 |
| Modelling Theory - Overview | 8 |
| Definitions | 8 |
| Types of Models for Even aged Stands | 9 |
| Models in Tabular Form | 10 |
| Normal yield tables | 10 |
| Empirical yield tables | 11 |
| Variable density growth and yield models | 12 |
| Explicit yield prediction models | 12 |
| Implicit yield estimation | 13 |
| Individual tree models | 14 |
| Modelling data | 14 |
| Data, Justification, Analysis Tools and procedures | 17 |
| Data Available and its justification | 17 |
| Analysis Tools and Procedures | 19 |
| Skewness | 20 |
| Kurtosis | 20 |
| Residual mean | 21 |
| Results | 22 |
| Mean top height and basal area growth | 22 |
| Mean top height | 22 |
| Basal Area | 24 |
| Differential Growth Functions | 25 |
| Height | 27 |
| Basal area | 28 |
| Volume Functions | 34 |
| Tree Volume Equations | 34 |
| Stand Volume equation | 35 |

| | |
|---|----|
| Mean Top height and basal area vs Stocking | 38 |
| Discussion | 41 |
| Basal area and mean top height modelling | 41 |
| Log Reciprocal and Hossfeld functions for h_{100} | 42 |
| Log Reciprocal and Hossfeld for G | 42 |
| Basal area and mean top height vs stocking. | 44 |
| Conclusions and Recommendations | 46 |
| Summary | 49 |
| Acknowledgements | 51 |
| References | 52 |
| Appendix | 57 |
| Appendix A. | 57 |
| Appendix B1. | 58 |
| Appendix B2. | 60 |
| Appendix C | 64 |

List of Figures

| | |
|--|----|
| Fig 1. The hierarchy of forest planning models (Modified from Temu, 1992) | 9 |
| Fig 2. Mean Top Height Growth for three trials | 23 |
| Fig 3. Cumulative Basal Area Growth for three trials | 24 |
| Fig 4. Residual and Frequency plots for height, log reciprocal function, fitted to non overlapping data | 30 |
| Fig 5. Residual and Frequency plots for height, log reciprocal function fitted to overlapping data | 30 |
| Fig 6. Residual and frequency plots for height, Hossfeld function fitted to non overlapping data | 31 |
| Fig 7. Residual and frequency plots for height, Hossfeld function fitted to overlapping data | 31 |
| Fig 8. Residual and frequency plots for basal area, log reciprocal for non overlapping data. | 32 |
| Fig 9. Residual and frequency plots for basal area, log reciprocal function fitted to overlapping data | 32 |
| Fig 10. Residual and frequency plots for basal area, Hossfeld function fitted to non overlapping data. | 33 |
| Fig 11. Residual and frequency plots for basal area, Hossfeld function fitted to overlapping data | 33 |
| Fig 12. Residual and frequency plots for Volume (tree volume data from all NZ eucalypts equation) | 37 |

| | |
|--|----|
| Fig 13. Residual and frequency plots for Volume (tree volume data from <i>E.regnans</i> equation) | 37 |
| Fig 14. Mean Top Height per Stocking for three trials | 39 |
| Fig 15. Basal Area per Stocking for three trials | 40 |
| Fig 16. Map of Zimbabwe showing the location of Eastern Highlands and Trial sites (not drawn to scale) | 57 |
| Fig 17. Residual and frequency plots for height, log reciprocal fitted to non overlapping data | 60 |
| Fig 18. Residual and frequency for height, log reciprocal function fitted to overlapping data. | 60 |
| Fig 19. Residual and frequency plots for height, Hossfeld function fitted to non overlapping data. | 61 |
| Fig 20. Residual and frequency plots for height, Hossfeld function fitted to overlapping data. | 61 |
| Fig 21. Residual and frequency plots for basal area, log reciprocal function fitted to non overlapping data. | 62 |
| Fig 22. Residual and frequency plots for basal area, log reciprocal function fitted to overlapping data | 62 |
| Fig 23. Residual and frequency plots for basal area, Hossfeld function fitted to non overlapping data. | 63 |
| Fig 24. Residual and frequency plots for basal area, Hossfeld function fitted to overlapping data (no initial stocking variable). | 63 |

List of Tables

| | |
|--|----|
| Table 1. Locational Climatic and Physical Characteristics. | 22 |
| Table 2. List of models and Data used (1 to 4 are log reciprocal functions and 5 to 8 are Hossfeld functions). | 26 |
| Table 3. Comparison of fitting indices for log reciprocal and Hossfeld functions fitted to non overlapping data. | 28 |
| Table 4. Comparison of fitting indices for log reciprocal and Hossfeld functions fitted to non overlapping data. | 28 |
| Table 5. Comparison of fitting indices for log reciprocal and Hossfeld functions fitted to overlapping data. | 29 |
| Table 6. Comparison of fitting indices for log reciprocal and Hossfeld functions fitted to overlapping data. | 29 |
| Table 7. Non-linear parameter estimates for the stand volume function (11). | 36 |
| Table 8. Statistics for the stand volume function (11) | 36 |
| Table 9 .Comparison of fitting indices for log reciprocal and Hossfeld functions fitted to non overlapping data. | 58 |
| Table 10. Comparison of fitting indices for log reciprocal and Hossfeld functions fitted to non overlapping data. | 58 |
| Table 11. Comparison of fitting indices for log reciprocal and Hossfeld functions fitted to overlapping data. | 59 |
| Table 12. Comparison of fitting indices for log reciprocal and Hossfeld functions fitted to overlapping data. | 59 |

| | |
|---|----|
| Table 13. Extract of the form in which the data was modelled. | 64 |
| Table 14. h_{100} and G estimates from log reciprocal and Hossfeld functions, with and without the stocking variable | 66 |
| Table 15. Prediction of volume at age 7, from G and h_{100} at age 4. | 66 |

Abstract

Data from Nelder spacing trials for a limited range of ages from 1 to 7, were used to model the growth of basal area and mean top height of *Eucalyptus grandis* in Zimbabwe. Two projection functions, the log reciprocal and Hossfeld were tested on data arranged as non overlapping ($T_1-T_2, T_2-T_3, \dots, T_{n-1}-T_n$) and overlapping ($T_1-T_2, T_1-T_3, T_1-T_n, T_2-T_3, T_2-T_n$, etc). The log reciprocal performed poorly on both overlapping and non overlapping mean top height (h_{100}) data. The Hossfeld function fit to overlapping and non overlapping h_{100} data did not appear to be very different, but overall the fit to overlapping data was slightly superior. The log reciprocal fitted both overlapping and non overlapping basal area (G) data better than it did with h_{100} data, but the Hossfeld still proved superior with the overlapping data. The functions were first run without the initial stocking variable and although its inclusion caused small improvements in the fitting. The small improvements that were used to eliminate some of the functions were quite important when the h_{100} and G were converted to volume/ha in the stand volume equation. The Hossfeld function, with the initial stocking included, fitted to overlapping data for both h_{100} and G , was the preferred choice of projection equation.

Introduction

Forestry in Zimbabwe

Zimbabwe has a total land area of 39 million ha, of which 23 million ha are in forest

including 22 million ha of unmanaged natural forest, 900 000 ha of managed natural forest and 104 590 ha of plantations (Forestry Commission survey, 1990, and 14th Commonwealth Conference, 1993). Thus, less than 1% of Zimbabwe's land area is covered in forest plantations, the bulk of which are in the Eastern Highlands (see map Fig 16 Appendix A) where rainfall, altitude and soil are quite favourable for tree growth. *Pinus patula*, *P. elliottii* and *P. taeda* dominate the coniferous species. Pine species are principally grown for commercial purposes while Eucalypts, second in importance to pine, serve both commercial and with multipurpose rural needs. Pines were first introduced in the late 1920s and large scale plantings took place after the Second World War in the Eastern Highlands to prepare the country for anticipated future demand for softwood timber. Zimbabwe does not have its own indigenous softwoods for commercial use (Kanyemba 1984). *Pinus radiata*, which is the golden species in New Zealand and Chile (among the world's largest radiata growers) was among the first species to be tried, but it was severely hit by the fungal pathogen *Diplodia* and was never pursued. Pine, *Eucalyptus* and black wattle (*Acacia mearnsii*) together provide the bulk of Zimbabwe's timber requirements. Production is estimated to be 600 000 m³ sawlogs, 33 000 m³ veneer logs, 94 000 m³ pulpwood, 80 000 m³ small roundwood for mining timber and 37 000m³ for fuelwood and charcoal (Forestry Commission (Zimbabwe), 14th Commonwealth Conference, 1993). It has been suggested that the country is underutilising the resource, only 33% of the total annual increment from plantations being utilised (Burley *et al.*, 1989).

There are very few commercially significant indigenous trees in Zimbabwe. The vast majority of the country's indigenous resource is woodland or scattered bush. The woodlands have been subjected to sustained destruction, for firewood, cattle grazing,

and food cropping, particularly since the forced relocation of the local people following the arrival of European settlers.

Woodland tree species vary from one part of the country to another. Above 1200 m, in the eastern part of the country, broadleaf species, msasa (*Brachystegia spiciformis*) and mnondo (*Julbernardia globiflora*) dominate; below 1200 m *mfuti* (*B. boehmii*) is most abundant. In the western dry areas below 1200 m, mangwe (*Terminalia servicea*) is most commonly found and in the Zambezi valley below 900 m, the dominant species are mopane (*Colophospermum mopane*) and baobab (*Adansonia digitata*).

The Forestry Commission is responsible for sustained harvesting of the indigenous trees. Currently they are giving concessions to local authorities and providing technical support. However, not many of the natural forest trees are commercially useful and their sparse distribution makes gainful harvesting difficult. Those that are exploited include mukwa (*Pterocarpus angolensis*), Zimbabwean teak (*Baobab plurijuga*) and mahogany (*Guibourtia coleosperma*). Harvesting operations for the latter group of species are confined to Matebeleland North. Emphasis is given here to the analysis involved in trying to refine appropriate systems of modelling growth and yield of *Eucalyptus* plantation species.

The History of Eucalypts in Zimbabwe

Eucalyptus species which found their way to Zimbabwe from their native Australia via South Africa (Barret and Mullin, 1968), are the most widely grown exotic hardwood species in Zimbabwe, mainly because of their diverse utilities, fast growth and their

ability to adapt to a wide range of environments. The earliest Eucalypt species (*Eucalyptus grandis*) was introduced into Zimbabwe in 1892 by the Meikle brothers (Radford, 1986) in Penhalonga. Planting increased through to 1923 and the first sawmill was built in the same year. Since then *Eucalyptus* plantations have spread throughout the country and by 1984, they comprised 19% of the 97 140 ha of commercial plantations (Kanyemba, 1984). Currently they account for 15% of the total forest plantation area (Zimbabwe Forestry Commission survey, 1990). The figure is lower than the 1984 figure not because of a planting/harvesting imbalance, but because the area planted in *Eucalyptus* has not increased to the same extent as other species. Nevertheless, there is recognition of a growing importance for Eucalypts in Zimbabwe, reflected in the establishment of a Eucalypt breeding programme by the research division of Forestry Commission and the widespread planting that has taken place in rural areas since independence in 1980. The breeding programme is responsible for selecting plus trees and by 1989, 382 plus trees had been selected and half-sib progeny tests have already been established (Burley *et al.*, 1989). The information given so far seems to indicate a *Eucalyptus* success story but it must be borne in mind that there are some strong reservations within the worldwide scientific community about committing large areas to Eucalypts. Some people believe that Eucalypts are ecologically damaging and others disagree. The literature for and against this viewpoint is too voluminous to present here (see FAO, 1985). Nonetheless, *Eucalyptus* trees have several desirable features which make them attractive for either forest plantations or small rural woodlots.

1. They are fast growing, with records of mean annual increment in volume of $66.4 \text{ m}^3 \text{ ha}^{-1} \text{ an}^{-1}$ for the fastest growing exotic hardwood in Zimbabwe, *E.saligna* compared to the average for Eucalypts of $17.5 \text{ m}^3 \text{ ha}^{-1} \text{ an}^{-1}$ (Barret and Mullin 1968). Because of this fast growth they can serve a range of purposes at only 10 years of age.
2. Wide variety of uses, fuelwood, poles, sawn timber, particle board, mining timber, charcoal and pulp.
3. Good coppicing reduces the need for artificial regeneration except for improved strains or changes of species.
4. Wide range of conditions under which the various species can grow.
5. Relatively easy to manage compared to other exotic species e.g self pruning.
6. Straight bole for most of the *Eucalyptus* species used.

The growing role of Eucalypts as a probable solution to the rapidly increasing demand for timber resources in Zimbabwe should be accompanied by collection and analysis of data on the important properties of Eucalypts including their growth and yielding capabilities. There is a real need to increase the tempo of development and revision of growth models to help quantify the benefits of selected breeds in terms of volume,

basal area and/or height. The same models could then also be applied to future and existing *Eucalyptus* plantations in Zimbabwe.

Modelling

Besides commercial plantations, Eucalypts in Zimbabwe have dominated the large scale rural afforestation programme initiated by the government after independence. It is the task of the research division of Forestry Commission to develop production models for plantations. Currently this is being done under the management trials programme which encompasses spacing trials, pruning trials, thinning trials and volume estimation (Burley, 1989). Until recently volume tables have been the only tool available to forest managers to assist in management decisions. Mathematical growth models are just beginning to appear for the most important species.

Data collection started with the beginning of plantation forestry back in the 1900's (Radford, 1987), but the research division of the Forestry Commission with the technical ability to analyze and make these data useful was not established until 1948. The Forest Research Centre now has models for *E. grandis*, *E. cloeziana* and *P. patula* (Crockford), built on data from spacing trials. Trials may not be the most desirable sources of modelling data, but they can be used for preliminary modelling, in the absence of the preferred data sources. Permanent sample plots in crops which are grown and treated just like any other plantation tree in the day to day management activities are widely recommended for growth modelling. On the one hand, a drawback of trial data is that the research treatment that trials get is not reproduced in day to day forest management; hence, the results may be unrepresentative of the

existing plantations. On the other hand, it is important in research trials to minimise confounding effects and apply local control.

Forest growth modelling in Zimbabwe could be said to be at an early stage of development with some reliance on expatriate experts and it is no exaggeration to say there is need to train locals in this area and/or related fields.

Objectives of the Study

The general objective is to gain some training in the field of growth and yield modelling which will be achieved by recognising the following specific objectives.

1. To develop equations for predicting growth in mean top height and basal area/ha for *E.grandis* which are applicable to the Zimbabwean situation.
2. To determine the influence of stocking on mean top height and basal area growth.

The second objective is secondary to the first one and it will not be covered to the same extent.

Literature Review

Modelling Theory - Overview

Definitions

A model is generally defined as *a mathematical or physical system obeying specified conditions, the behaviour of which is used to understand a physical, biological or social system and to which it is analogous in some way* (Ralston and Meek 1976). For forestry, a model is defined similarly as *a mathematical function, or system of functions, used to characterise actual growth rates for measured tree, stand and site variables* (Bruce and Wensel 1987). Models can be quantitative, qualitative or both; qualitative ones tend to be subjective and so less robust.

Growth and yield models need to be quantitative if they are to assist in objective decision making by forest managers; for example, quantifying growth responses of silvicultural treatments and for forecasting harvest yields. Growth and yield models can be used to predict current or future resource conditions in terms of various measures which include biomass, volume, basal area and stocking per unit area. Such predictions and forecasts are required for production planning, silvicultural research, ecological research and environmental management. Equations like the ones reported here are designed to serve production planning uses primarily.

Some authors prefer to call a system of mathematical growth functions collectively a model, while others recognise each individual function in the system as a model. In this report the latter case is assumed. Growth models and some uses for them are depicted in Fig 1.

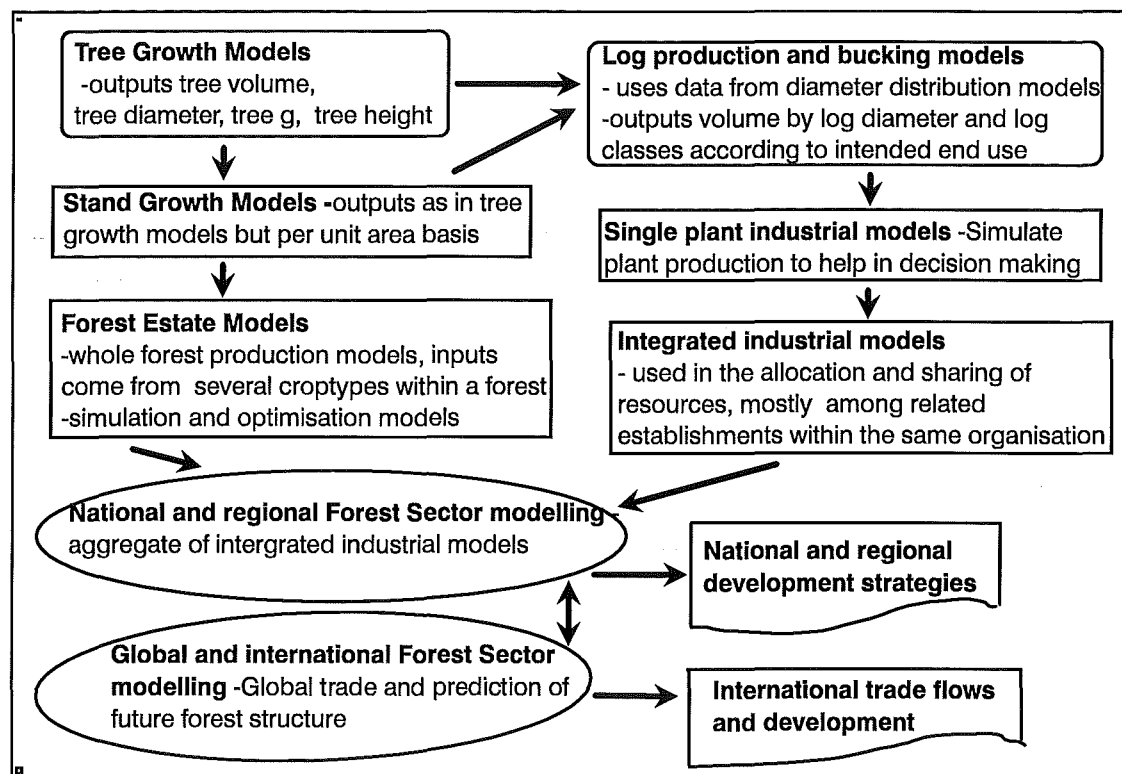


Fig 1. The hierarchy of forest planning models (adapted from Temu, 1992)

Fig 1. clearly shows that other models are directly or indirectly dependent on tree growth and stand models. The subject of this report is stand growth modelling of even aged *Eucalyptus grandis* in Zimbabwe, the implied representation in the second box down on the left hand side of Fig 1.

Types of Models for Even aged Stands

Clutter *et al.*, (1983), classified growth models for even aged forest stands as follows;

A. Models in Tabular Form

B. Models as equations and systems of equations

1. Direct prediction of unit area values
2. Unit area values obtained by summation
 - i. Equations for classes of trees
 - ii. Equations for individual trees

B(1) refers to the earliest variable density Schumacher type yield models and most other multilinear and non-linear functions. B(2)(i) incorporates diameter distribution models and B(2)(ii), distance dependent and distance independent models.

Avery and Burkhart (1983) have a very similar classification.

1. Normal yield tables
2. Empirical yield tables
3. Variable density growth and yield equations
4. Size class distribution models
5. Individual tree models

These classes are now discussed under a set of headings which combine both classifications.

Models in Tabular Form

a. Normal Yield tables

These were 19th Century methods designed in Germany based on data obtained from what were called normal forest stands (Clutter *et al.*, 1983), or normal age class

distribution where every age class up to rotation age was required to be fully stocked. Striving for so called normality and full stocking, in each stand was all to do with continuity of timber supply. Graphs would then be drawn of volume for each age class through to rotation age, separately for each of various crop productivity classes. Stock tables were then constructed by reading volume for each age class off the graphs.

The USA adaptation of normal yield tables included measures of site quality to produce tables of volume per given age class and site quality. Stocking was still held constant at a fixed level, which was one of the drawbacks that encouraged the development of better systems. Yield tables of this type also contain auxiliary information such as basal area/ha (G), stocking/ha (N), diameter distributions and volume/ha (Avery and Burkhart, 1983). The other drawbacks of the normality concept were: a separate table for every different stocking density class was needed, a cumbersome and costly requirement; the normal forest concept bears little relation to reality.

b. Empirical yield tables

These tables were meant to overcome the stocking limitation of normal yield tables. They were developed from stands with average stocking rather than full stocking but their application was still limited to stands with average stocking. This further encouraged modern day models that incorporate stocking or some other measure of stand density as a variable, and which do not need to be tied to the normal forest concept.

Variable density growth and yield equations

a. Explicit Yield prediction Models

Explicit growth models predict variables such as volume/ha (V), basal area/ha (G), and mean top height (h_{100}). The models comprise a system of equations where outputs from one are the inputs to the other as in the examples given below.

Overall Yield equations

$$\ln V = \beta_0 + \beta_1 T^{-1} + \beta_2 f(S) + \beta_3 G \quad (i)$$

Where:

V = stand volume in m³/ha

S = site Index (height of dominant trees at an index age)

T = stand age in years

G = basal area m²/ha

β = regression parameters estimated by linear least-squares

Equation (i) is a simple multilinear regression model of the Schumacher type. Such models have been used extensively in the past and even today, but they do not always represent stand volume growth very well and the currently preferred growth modelling approach is to forecast changes in more easily assessed variables such as h_{100} , G and N, before converting these to stand volume. This approach requires tree and stand volume functions like equation (ii) to be available.

$$V = \alpha G^\beta \times \bar{h}_{100}^\gamma \quad (ii)$$

Where α , β and γ = non-linear least squares regression parameters

Differential projection as in equations (iii) and (iv), can be used to characterise stand growth development over time.

Differential growth functions, the Hossfeld below in (iii) and log reciprocal in (iv) as set out in Woollons *et al.*, (1990).

$$Y_2 = 1 / ((T1/T2)^\beta \times 1/Y_1 + (\alpha) \times (1 - (T1/T2)^\beta)) \quad (iii)$$

$$Y_2 = Y_1^{(T1/T2)^\beta} \times \exp(\alpha(1 - (T1/T2)^\beta)) \quad (iv)$$

Where:

Y_1, Y_2 = are sample plot values of basal area (m²/ha) and mean top height at
T1 and T2

T1, T2 = age at measurement time 1 and measurement time 2

α, β = as in (ii)

Equation (ii) converts the basal area/ha and mean top height equations (iii) and (iv) to volume/ha. Differentiating the yield function with respect to T gives the growth function. Separating the Y and T variables in the growth function and integrating both sides, will give the projection equations as in (iii) and (iv) above (Villanueva, 1992).

b. Implicit Yield estimation

Diameter distribution models can utilise this same approach through predicting changes in the number of trees per unit area by dbh class, using probability density functions. The Weibull distribution has been widely used to estimate number of trees

in each class (Clutter *et al.*, 1983) and more recently the Reverse Weibull distribution (Liu Xu, 1990). Multilinear regression of height as a function of diameter and stocking are used to predict mean height for each dbh class which can then be combined to determine the volume by dbh class from the relevant tree volume equation. The volume of the average tree is then multiplied by the number of trees/ha in each class to get unit area values, and so a volume per ha is derived implicitly. Diameter distribution models are not utilised in this study, though they could have an important role to play for plantation forestry crops in Zimbabwe's in future.

c. Individual tree models

These model tree rather than unit area growth and so unit area values are implicitly the sum of the tree values. Distance dependent tree models use such variables as diameter at breast height, crown length, tree spacing and competition indices while distance independent models use the same variables except for competition indices. These are not, however, part of this report, as they provide too great a level of detail for forecasting short rotation plantation yields.

Modelling Data

The collection of data for modelling varies depending on the sort of model one intends to produce, the personnel and financial resources available and other intended uses for the data. The data used here were obtained from Nelder spacing trials, which modellers would rather avoid using for reasons that are still debatable (Schonau and Coetzee, 1989). A summary of what Moser and Hall, (1969), consider the ideal data for modelling is given below.

a. Real growth series - Complete chronological records of several stands from establishment to harvest. This has the disadvantage of having to wait for a long time before all the chronological stands are established, which is feasible but not a desirable option.

b. Abstract growth series - Data from numerous temporary sample plots covering a wide range of sites and ages. This imitates real growth series and avoids undue waiting. Such data can be arranged to ensure independence of the error terms, but are restricted to developing yield functions.

c. Approximated real growth series - Permanent sample plots that are measured at intervals up to the end of the rotation. The waiting is unavoidable although some of the data can still be usable before the end of the rotation. Clutter, 1962 used such data to derive compatible growth and yield model for loblolly pine, but there are different schools of thought concerning the validity of using remeasured data in regression models (Woollons and Hayward, 1985; Sullivan and Reynolds, 1976). The most contentious issue is that the errors in remeasured data are correlated, which negates one of the basic requirements of regression. Woollons and Hayward, 1983 argue however, that as long as ordinary statistical significance tests are not going to be used in the analysis, it is quite legitimate to use regression to model correlated data.

In practise the implicit models, which predict yields per unit area by summation are more precise than explicit models, which predict unit area values directly. In theory, implicit models based on individual trees would be the best but they tend to demand

so much time and resources that they are not used often. This study will focus on the variable density explicit yield models using data that was obtained from some Nelder trials. Nelder trial analysis has received a lot of attention over the years.

Data, Justification, Analysis Tools and procedures

Data Available and its justification

Eucalyptus grandis data from three Correlated Curve Trend (CCT) Nelder trials (cct32, cct47 and cct49), were obtained from the Forestry Research Centre, Highlands, Harare in Zimbabwe. The data were used here to develop functions for determining current or future stand mean top height and basal area. Tree volume equations and taper functions for this species were not available from the research centre and New Zealand ones were used instead. This last part of the report should be treated as purely theoretical until Zimbabwean equations replace the New Zealand ones.

Two of the trials are located in the Eastern Highlands where climate and altitude are favourable for tree growth and the third is located further inland (see Fig 16 Appendix A). The CCT concept was designed for spacing trials in South Africa in 1935 (Bredenkamp 1984, Burgers 1971) and adopted in Zimbabwe. The concept is based on three assumptions:

1. in any given locality the size attained by a tree of a particular species at a given age must be related to the growing space previously at its disposal, all other factors influencing its size being fixed by locality;
2. trees planted at a given spacing will, until they start competing with each other, exhibit the absolute or normal standard of growth for the species and locality;

3. trees planted at a given spacing and left to grow unthinned will exhibit the absolute or normal standard of growth for the species, locality and the particular density of stocking in question.

The CCT concept has been much criticised, one of the major criticisms being that different conclusions can be reached from the same set of data (Schonau, 1989). Although the type of conflicting conclusions were not specified in this article the effect of growing space per tree in a Nelder design does not differ from that of a similar tree in another design with the same amount of growing space. Therefore the reason for reaching different conclusions could be a result of the methods used to analyse the data rather than the data themselves.

The Nelder design (Nelder, 1962), and its use in spacing trials has also come under fire for a number of reasons concerning analysis of the results. One major issue is associated with how the stocking for each of the rings is worked out. One way of doing this is to relate the area occupied by each tree to a hectare and use simple proportions to derive the stocking. The area occupied by trees in a ring can also be related to a hectare in the same manner and stocking then derived. Another concern has been levelled at the loss of stems as this affects the remaining neighbouring trees. Similar losses exist in any experimental set up but do not render the data from the remainder null and void. Nelder data can still be used in modelling provided that adjustments are made properly to cater for unplanned irregularities in the spacing. The other area of disagreement has been whether to model data from all rings together or individually. In the case of permanent sample plots, modellers try to encompass as many stocking levels as possible so as to broaden the applicability of

the model. Such data are grouped and modelled together because the variation due to the different stockings can be included in the model. The same line of argument can be extended to the different rings of a Nelder trial. This also avoids the multiplicity of models that could result from modelling data from each ring as an entity. The concept of grouping data from different stockings and characterising it with a stocking variable can also be extended to data from different environments, identifying each environment by a dummy variable (Whyte *et al.*, 1992, Gujarati, 1970)

Analysis Tools and Procedures

A spreadsheet package, Quattro pro version 5.00 was used to sort out the raw data into the desired modelling format. Simple two axis graphical plots were used to identify obvious data outliers, the reliability of which could be checked from the data. G and h_{100} were also calculated in the spreadsheets. The data resulting from these procedures were transferred to SAS (Statistical Analysis Systems, SAS institute Inc. 1990) for further cleaning and then fitting models. Fitting linear and multilinear models is very easy and straightforward in SAS, but the nonlinear models used here proved quite challenging: specifying starting values for the regression parameters requires experience and understanding of how the model responds to certain initial parameters. There are several methods of estimating initial parameters; linearization, steepest descent and Marquardt's compromise (Draper and Smith, 1981), but they do not guarantee any success in convergence.

The *proc nlin* procedure was used to fit the differential growth functions for h_{100} and G , and the volume functions. This procedure outputs the parameters for the equations, their variation in terms of standard errors and mean square errors, and

residual plots. When the residuals are normally distributed they should be spread evenly about the zero reference line for a range of variations on the x-axis. Bar charts, stem leaf plots and normality plots add to the visual assessment for fitting models. These visual tools provide a very useful way for either further eliminating outliers and/or judging the goodness of fit of the models.

The *proc univariate* procedure outputs information about the residuals which further helps in determining the goodness of fit of chosen models. The outputs are discussed under the headings below.

Skewness

Skewness measures the tendency of the residual deviations from their mean to be larger in one direction than the other (SAS institute Inc. 1987). Sample skewness is calculated in SAS as:

$$\frac{n}{(n-1)(n-2)} \times \sum_{i=1}^n \frac{(x_i - \bar{x})^3}{s^3}$$

The value of skewness can be positive or negative and is unbounded. An ideal distribution has skewness zero and in this case if the value was far removed from zero the fitting was reassessed.

Kurtosis

Kurtosis is a measure of the heaviness of the tails of the distribution of the residuals and it is not desirable to have too many or too few data in the tails. In such cases a reassessment of the fitting of the model is called for. Population kurtosis must lie between -2 and positive infinity inclusive (SAS Institute Inc. 1987). Sample kurtosis

in SAS is calculated as:

$$\frac{n(n+1)}{(n-1)(n-2)(n-3)} \times \frac{\sum_{i=1}^n (x_i - \bar{x})^4}{s^4 - 3(n-1)(n-2)(n-3)}$$

Residual mean

The residual mean should be close to zero for the residuals to be close to a normal distribution.

Interpretation in this way of *Proc nlin* and *proc univariate* outputs together provide a powerful tool for model selection.

Results

Mean Top Height and Basal Area Growth

Mean top height

Mean Top height (h_{100}), increases with time at different rates for the three trials, as shown in Figs 1 (a), (b) and (c). It is important to emphasize that Figs 1 (a) and (c) are based on limited data, because measurements prior to ages 4 and 5 were not available. Comparisons can however be made from ages 5 to 7, for which data were available in all three trials. The wide gap in height growth between ages 4 and 5 for trial cct47 could be due to an error in measurement which makes it impossible to determine the exact path of height growth prior to age 5 or immediately after age 4, but the data were retained and included in the analyses. The height growth differences in the three trials reflect variation due to location, part of which is contributed by the mix of environmental features, as shown in Table 1.

Table 1. Locational Climatic and Physical Characteristics.

| Place/Trial | Altitude (m) | Latitude | Longitude | Mean annual Rainfall (mm) | Mean annual Temp(°C) |
|-----------------------|-----------------|----------|-----------|---------------------------------|-------------------------|
| Mtao (cct32) | 1477 | 19° 22' | 30° 38' | 755.5 | 17.7 |
| Gungunyana (cct47) | 1050 | 20° 24' | 32° 43' | 1409.7 | 18.1 |
| Muguzo (cct49) | 1483 | 19° 54' | 32° 53' | 1409.8 | 16.6 |

Mean Top Height

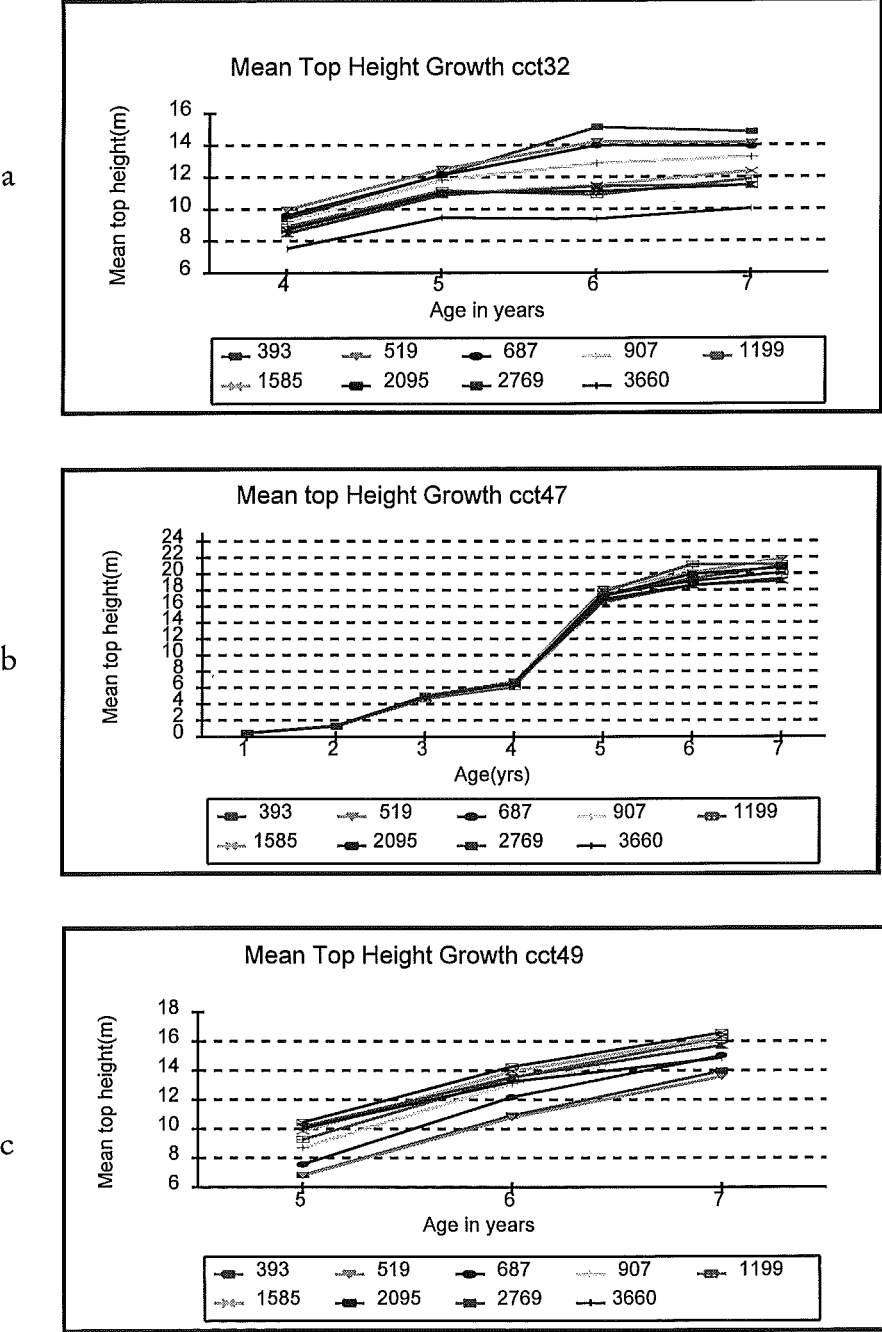


Fig 2. Mean Top Height Growth for three trials

Basal Area

Stem mortality is quite high in some cases and resulted in negative growth, which is clearly shown in trial cct32 for most of the stocking levels and in cct47 for one of the stocking levels (see Fig 2 a and b).

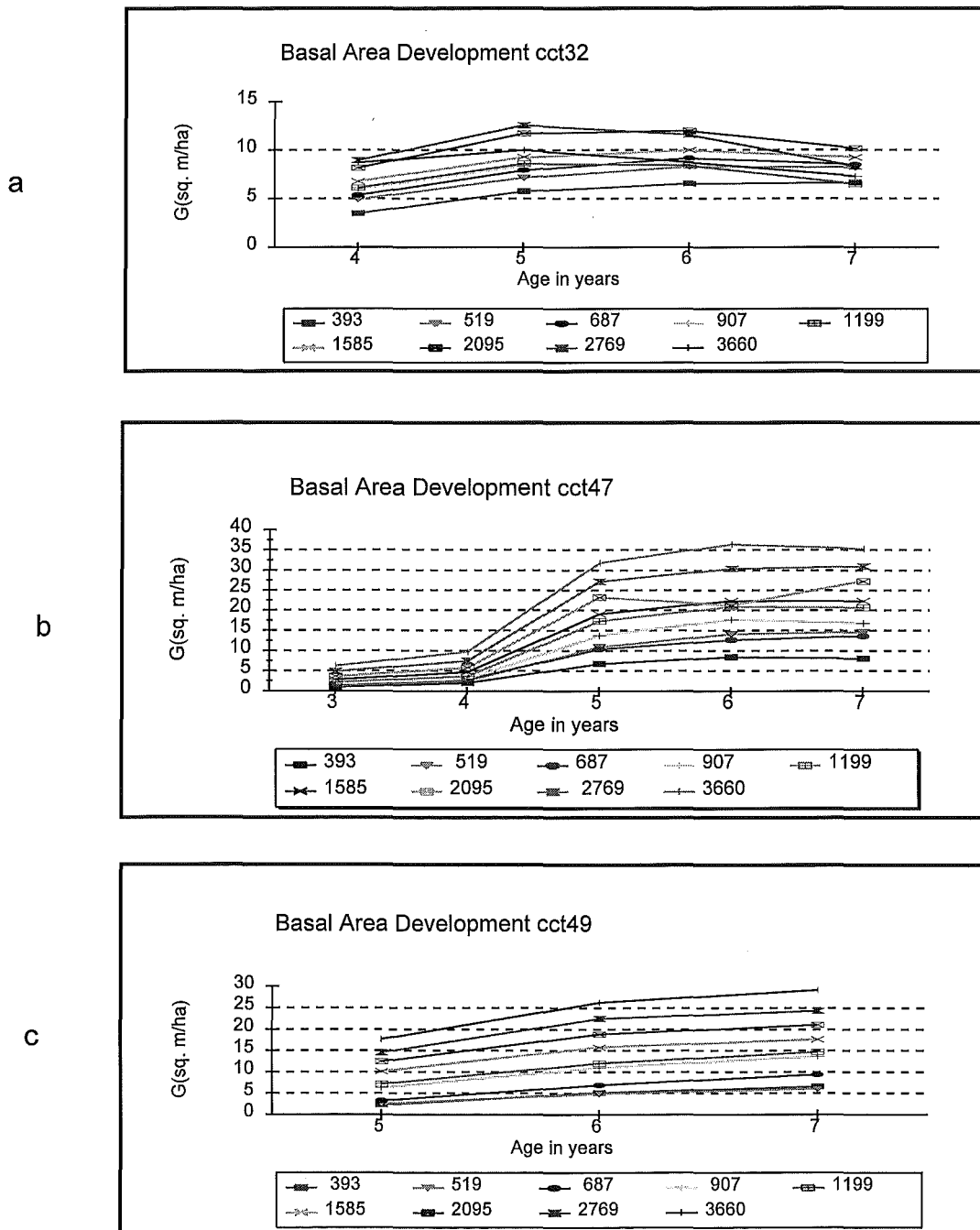


Fig 3. Cumulative Basal Area Growth for three trials

Differential Growth Functions

Because of the broad similarity in growth trends, the data for all three trials were pooled for basal area and h_{100} growth modelling. Variation due to stocking differences is represented in the models, but locational variation was not, even though it has the potential to improve the models. This can be done by including data from the various climatic and environmental factors of each area as extra variables in the model. Such data were not available in this case. Dummy variables could have been used as an alternative but they were not included because the quantity and quality of the available data preclude at this stage the advisability of this sensitive approach.

The log reciprocal and Hossfeld functions were used to model the growth of G and h_{100} from the three trials. Models were run for the following sequences shown below, where non overlapping refers to the intervals; $T_1-T_2, T_2-T_3, \dots, T_{n-1}-T_n$ and overlapping refers to intervals; $T_1-T_2, T_1-T_3, T_1-T_4, \dots, T_1-T_n, T_2-T_3, T_2-T_4, \dots, T_2-T_n$, etc., Table 2.

Table 2. List of models and Data used (1 to 4 are log reciprocal functions and 5 to 8 are Hossfeld functions).

| | Data Used | Stocking Variable | Equation |
|---|-----------------|-------------------|--|
| 1 | non overlapping | not included | $Y_2 = Y_1^{(T1/T2)^\beta} \times \exp(\alpha(1 - (T1/T2)^\beta))$ |
| 2 | non overlapping | included | $Y_2 = Y_1^{(T1/T2)^{\beta+\gamma \times N}} \times \exp(\alpha(1 - (T1/T2)^{\beta+\gamma \times N}))$ |
| 3 | overlapping | not included | $Y_2 = Y_1^{(T1/T2)^\beta} \times \exp(\alpha(1 - (T1/T2)^\beta))$ |
| 4 | overlapping | included | $Y_2 = Y_1^{(T1/T2)^{\beta+\gamma \times N}} \times \exp(\alpha(1 - (T1/T2)^{\beta+\gamma \times N}))$ |
| 5 | non overlapping | not included | $Y_2 = 1 / ((T1/T2)^\beta \times 1/Y_1 + (\alpha) \times (1 - (T1/T2)^\beta))$ |
| 6 | non overlapping | included | $Y_2 = 1 / ((T1/T2)^{\beta+\gamma \times N} \times 1/Y_1 + (\alpha) \times (1 - (T1/T2)^{\beta+\gamma \times N}))$ |
| 7 | overlapping | not included | $Y_2 = 1 / ((T1/T2)^\beta \times 1/Y_1 + (\alpha) \times (1 - (T1/T2)^\beta))$ |
| 8 | overlapping | included | $Y_2 = 1 / ((T1/T2)^{\beta+\gamma \times N} \times 1/Y_1 + (\alpha) \times (1 - (T1/T2)^{\beta+\gamma \times N}))$ |

Where: Y = Either G or h_{100}

T = Age in years

N = initial stocking

α , β , γ = Non-linear regression parameters

The goodness of fit of each of the equations was evaluated on the basis of trends in residuals($Y_{obs}-Y_{pred}$) plotted on Y_{pred} and on skewness, kurtosis and mean deviation of the residuals from zero. In addition other indicators such as residual sums of squares(ESS), mean square error(MSE) and standard error of estimate(SEE) were also taken into account.

The models were first run without the initial stocking variable, and when this variable was included the improvement was not very great, nevertheless, this small improvement made a substantial difference when the projected G and h_{100} were converted to volume by the volume function. A worked example of this is shown in Appendix C. Therefore only the results of models which included the initial stocking variable are presented here, and those of the same models without the initial stocking variable are shown in Appendix B, Tables 7 to 10 and Figs 17 to 24 .

Height

Log reciprocal function

The model appears not to perform well with either overlapping or non overlapping data as can be seen from Tables 3, 4, 5 and 6 and as represented visually in plots of residuals and their frequencies in Figs 4 and 5.

Hossfeld function

This model is better than the log reciprocal and it performs better with overlapping than with non overlapping data. Better here means in terms of kurtosis, skewness, SEE, mean residual, and residual plots. This is shown in Tables 3, 4, 5 and 6 and Figs 6 and 7.

Basal area

Log reciprocal and Hossfeld functions

The log reciprocal and Hossfeld functions fit relatively well to non overlapping data, Figs 8 and 10, but the fit improves with overlapping data as shown in Figs 9 and 11 and Tables 3, 4, 5 and 6.

The Hossfeld function for G and h_{100} , with the initial stocking variable included and fitted to overlapping data was judged to be superior to the others that were tried.

Table 3. Comparison of fitting indices for log reciprocal and Hossfeld functions fitted to non overlapping data.

| Equation | Parameter | Estimate | SEE | ESS | MSE |
|-------------------------------------|-----------|--------------|---------------|------------|----------|
| Log Reciprocal (h_{100}) (2) | α | 3.688021337 | 0.17662187942 | 154.666407 | 1.777775 |
| | β | 0.814725775 | 0.12620429399 | | |
| | γ | -0.000061540 | 0.00004599167 | | |
| Hossfeld (h_{100}) (6) | α | 0.042815200 | 0.00274467274 | 149.653434 | 1.720154 |
| | β | 2.487608025 | 0.27477708570 | | |
| | γ | -0.000238087 | 0.00013231629 | | |
| Log Reciprocal (G) (2) | α | 3.969833957 | 0.39750782834 | 365.976758 | 5.304011 |
| | β | 0.595230613 | 0.20522582073 | | |
| | γ | 0.000030387 | 0.00009710013 | | |
| Hossfeld (G) (6) | α | 0.025044992 | 0.00467325884 | 370.819065 | 5.374189 |
| | β | 1.297367558 | 0.40571566744 | | |
| | γ | 0.000079023 | 0.00020794328 | | |

Table 4. Comparison of fitting indices for log reciprocal and Hossfeld functions fitted to non overlapping data.

| Equation | Skewness | Kurtosis | Mean Residual | n |
|---------------------------------|----------|----------|---------------|----|
| Log Reciprocal (h_{100})(2) | 0.487593 | -0.52293 | 0.080181 | 90 |
| Hossfeld (h_{100}) (6) | 0.276385 | -0.62275 | 0.074141 | 90 |
| Log Reciprocal (G) (2) | 0.418319 | 0.782208 | -0.03557 | 72 |
| Hossfeld (G) (6) | 0.21623 | 0.519463 | 0.029294 | 72 |

Table 5. Comparison of fitting indices for log reciprocal and Hossfeld functions fitted to overlapping data.

| Equation | Parameter | Estimate | SEE | ESS | MSE |
|----------------------------------|-----------|--------------|-------------|------------|-----------|
| Log Reciprocal (h_{100}) (4) | α | 3.750351293 | 0.147950851 | 4383.88495 | 16.419045 |
| | β | 1.086327224 | 0.107121556 | | |
| | γ | -0.000029799 | 0.000018405 | | |
| Hossfeld (h_{100}) (8) | α | 0.041699173 | 0.001844272 | 3301.64477 | 12.365711 |
| | β | 3.271726626 | 0.139961295 | | |
| | γ | -0.000097438 | 0.000053597 | | |
| Log Reciprocal (G) (4) | α | 3.380938971 | 0.094889619 | 5556.30515 | 33.073245 |
| | β | 1.159447189 | 0.216368893 | | |
| | γ | 0.000690621 | 0.000239508 | | |
| Hossfeld (G) (8) | α | 0.038124057 | 0.002678160 | 6364.15669 | 37.881885 |
| | β | 2.779416568 | 0.416331029 | | |
| | γ | 0.000837408 | 0.000365549 | | |

Table 6. Comparison of fitting indices for log reciprocal and Hossfeld functions fitted to overlapping data.

| Equation | Skewness | Kurtosis | Mean | n |
|----------------------------------|----------|----------|----------|-----|
| Log Reciprocal (h_{100}) (4) | 0.270254 | -0.23358 | 0.251305 | 270 |
| Hossfeld (h_{100}) (8) | 0.102275 | -0.38357 | -0.01051 | 270 |
| Log Reciprocal (G) (4) | -0.17031 | 0.38654 | -0.3407 | 171 |
| Hossfeld (G) (8) | -0.11855 | -0.09262 | -0.21827 | 171 |

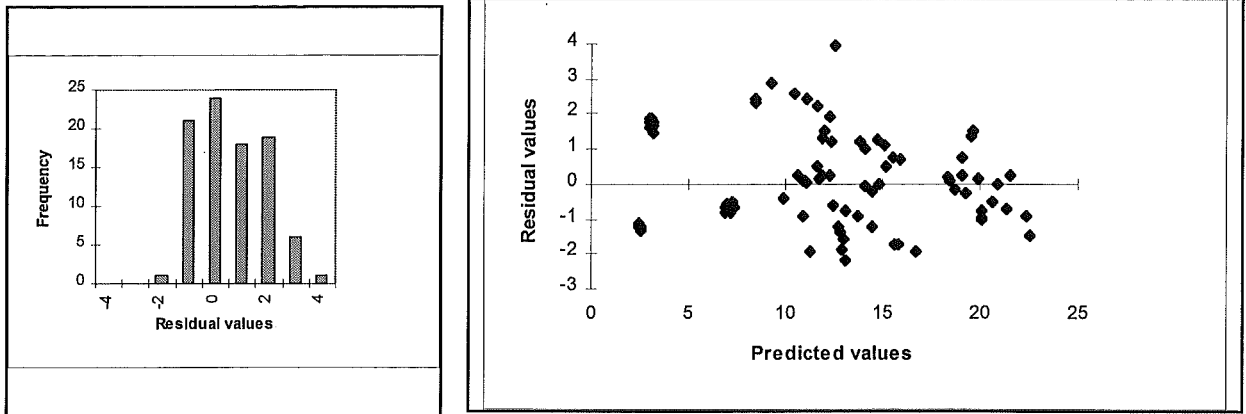


Fig 4. Residual and Frequency plots for height, log reciprocal function, fitted to non overlapping data

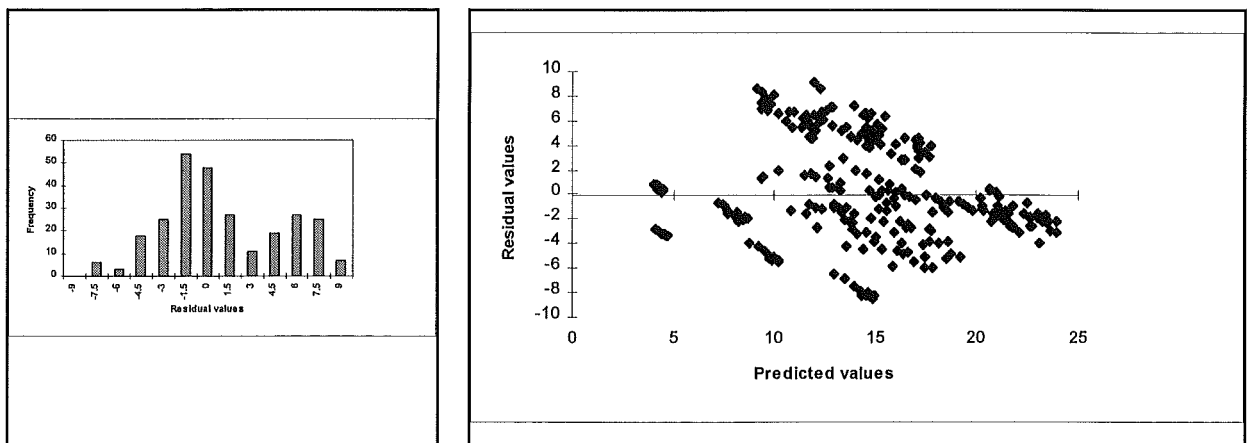


Fig 5. Residual and Frequency plots for height, log reciprocal function fitted to overlapping data

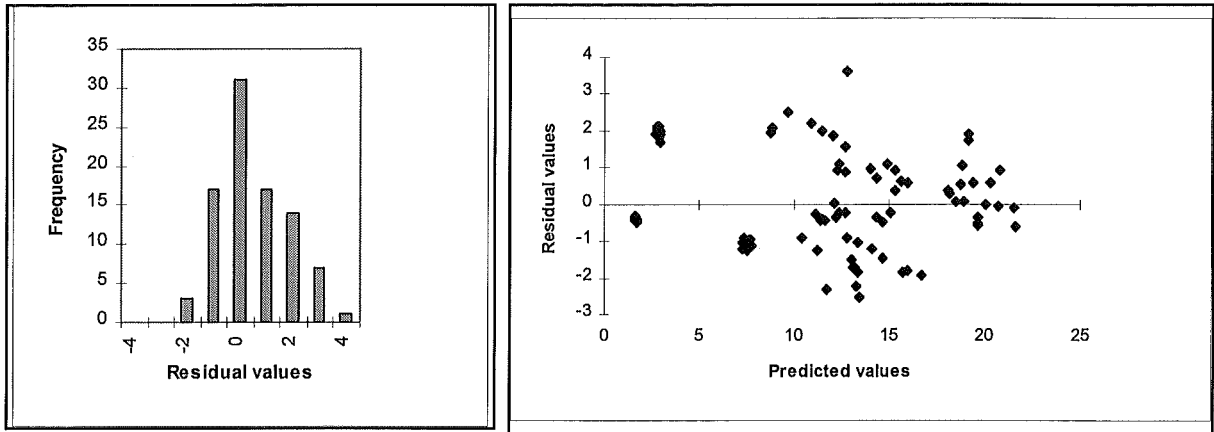


Fig 6. Residual and frequency plots for height, Hossfeld function fitted to non overlapping data

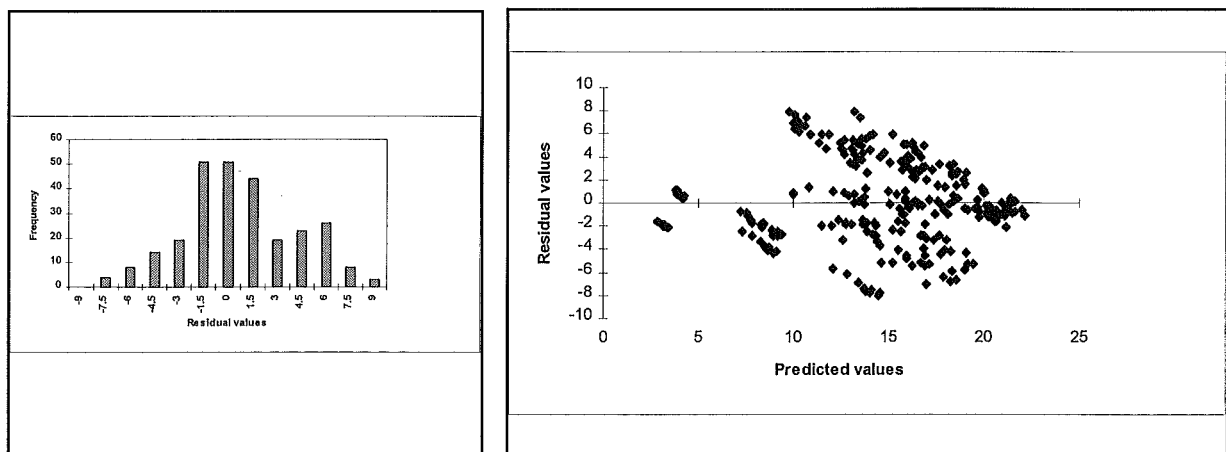


Fig 7. Residual and frequency plots for height, Hossfeld function fitted to overlapping data

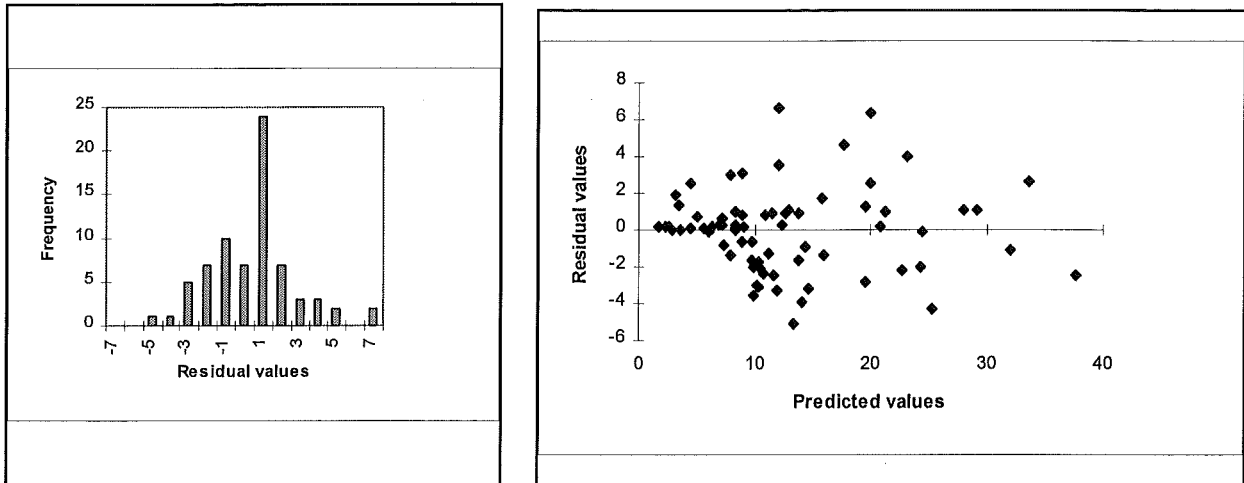


Fig 8. Residual and frequency plots for basal area, log reciprocal for non overlapping data.

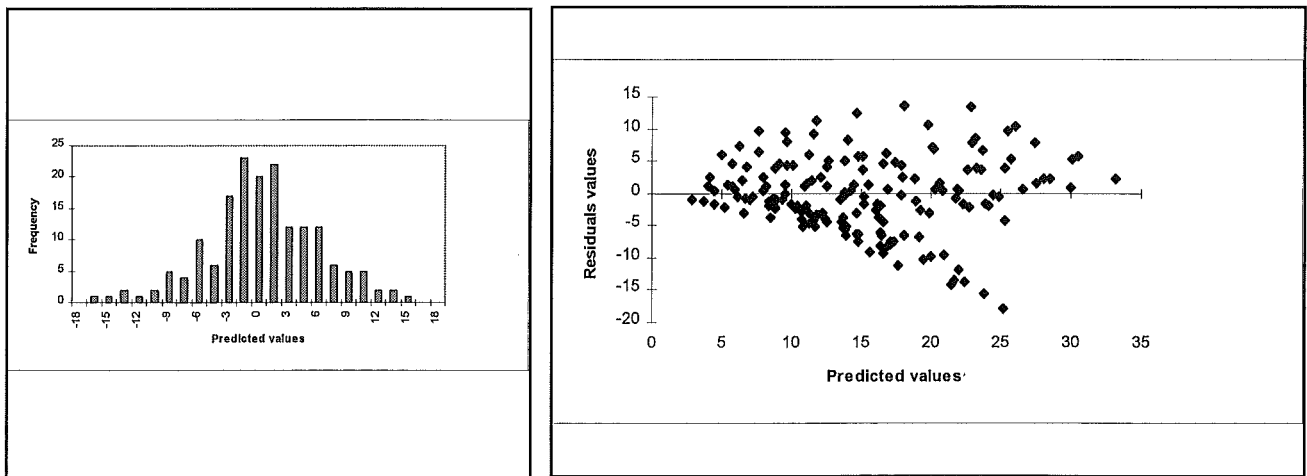


Fig 9. Residual and frequency plots for basal area, log reciprocal function fitted to overlapping data

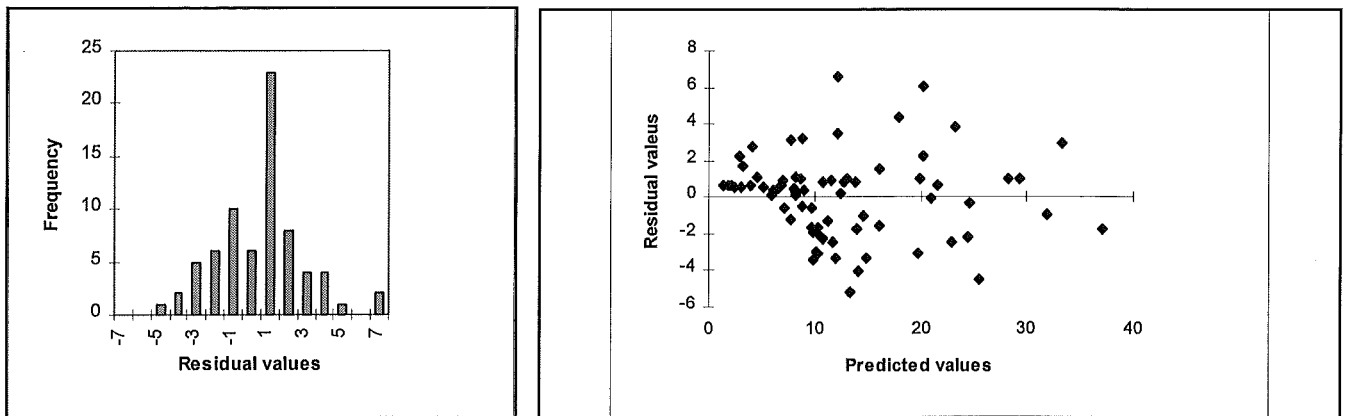


Fig 10. Residual and frequency plots for basal area, Hossfeld function fitted to non overlapping data.

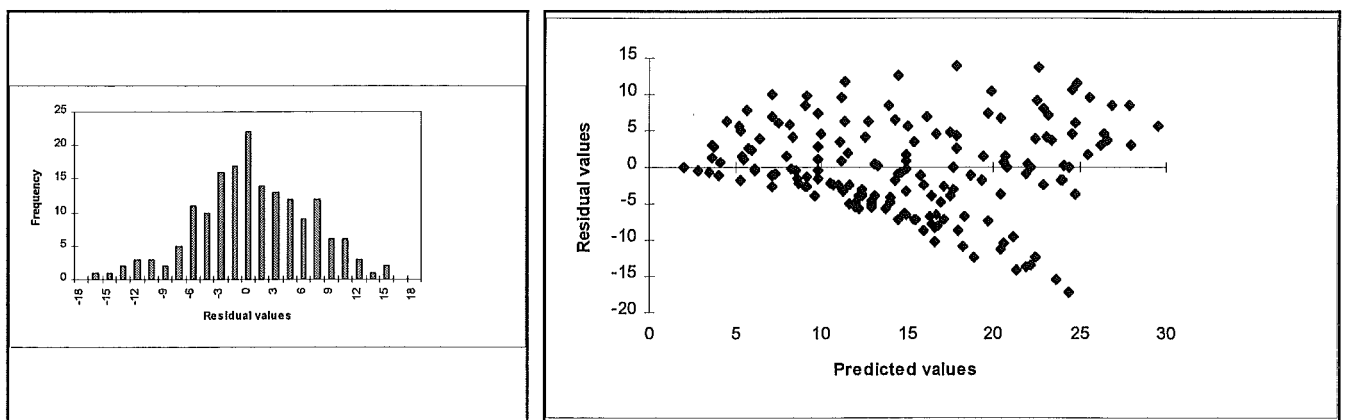


Fig 11. Residual and frequency plots for basal area, Hossfeld function fitted to overlapping data

Volume Functions

Tree Volume Equations

Sample plot measurements of height and diameter were converted into tree volumes. Zimbabwean data for developing tree volume equations were not available so New Zealand and South African equations were examined to see what might be substituted meanwhile.

South African equation

$$v = -4.2328 + 1.7154 \times \log(d-2) + 1.107 \times \log(h) \quad (9)$$

Where v = volume in m^3

d = diameter breast height (cm)

h = tree height(m)

For all New Zealand Eucalyptus

$$v = d^{2.009} \times (h^2 / (h - 1.4))^{7.57} \times \exp(-9.703557) \quad (10)$$

For NZ Eucalyptus regnans, Central North Island

$$v = 0.2984 \times d^2 \times h / 10000 \quad (11)$$

The South African equation could not cover the diameter range of the data used, so it was dropped from the analysis. In this case one would be inclined to take equation 10 as the appropriate one because it was built from several Eucalypt species, but

without knowing how the data for *E.regnans* compares to *E.grandis* this may not be a correct judgement. Therefore in this case either of these two can be used.

Stand Volume equations

Sample plot volumes were derived through aggregating the tree volumes calculated from the two New Zealand functions. These plot volumes expressed on a per hectare basis were then regressed on the two equation forms below.

Non-linear

$$v = \alpha G^{\beta} \times \bar{h}_{100}^{\gamma} \quad (12)$$

Where v = plot volume in m³/ha

\bar{h}_{100} = mean top height(m)

G = plot basal area in m²/ha

α, β, γ = non linear parameters

Linear

$$v = \beta_1 G + \beta_2 \bar{h}_{100} + \beta_3 N \quad (13)$$

The stand volume function parameters were derived from tree volume/ha data calculated using either of the NZ tree volume equations (10) and (11). The plot of residuals for equation (13) was not satisfactory, so stand volume equation (12), the pattern of residuals for which was superior, was used as the functional form. The parameters and goodness of fit are shown in Tables 7 and 8 and Figs 12 and 13 below.

Table 7. Non-linear parameter estimates for the stand volume function (11).

| Tree Volume Data Source | Para- meter | Estimate | SEE | ESS | MSE |
|---------------------------------------|----------------|-------------|---------------|-----------|----------|
| Data from all NZ <i>Eucalyptus</i> | α | 1.05324656 | 0.04451838955 | 992.8521 | 9.5467 |
| | β | 1.030916482 | 0.00735630363 | | |
| | γ | 0.650151091 | 0.01615602195 | | |
| Data from NZ <i>E.regnans</i> | α | 0.444056647 | 0.02304458692 | 1129.9102 | 10.86452 |
| | β | 1.052811778 | 0.00861214921 | | |
| | γ | 0.918250677 | 0.01985322612 | | |

Table 8. Statistics for the stand volume function (11)

| Tree Volume Data Source | Skewness | Kurtosis | Mean Residual | n |
|---------------------------------------|----------|----------|------------------|-----|
| Data from all NZ <i>Eucalyptus</i> | -0.78454 | 10.58014 | 0.150554 | 107 |
| Data from NZ <i>E.regnans</i> | -0.13236 | 3.949848 | 0.064105 | 107 |

Although the MSE for all New Zealand eucalypts is lower than for the *E. regnans*, the residual patterns are much better for the latter.

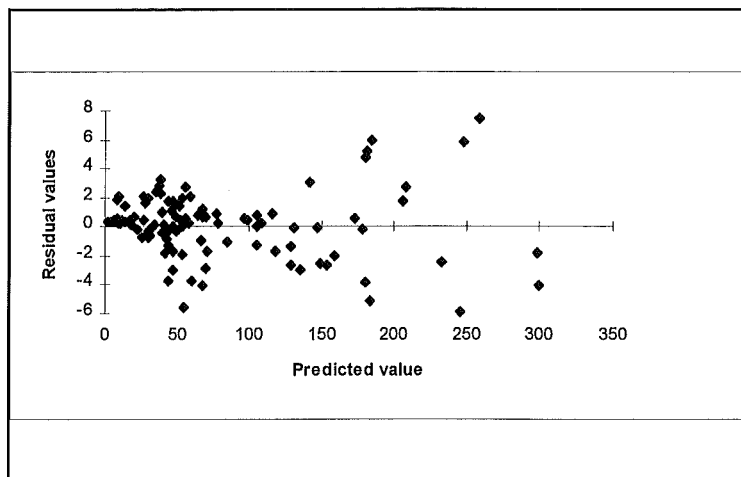
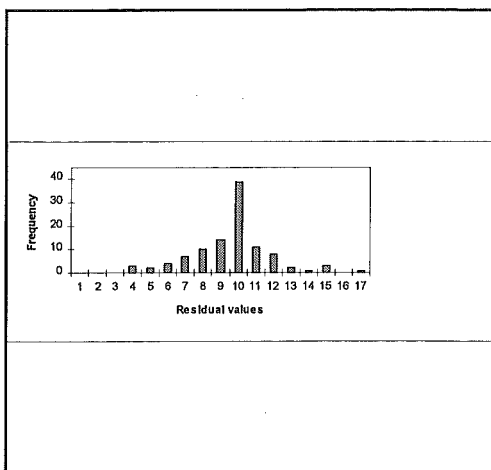


Fig 12. Residual and frequency plots for Volume (tree volume data from all NZ eucalypts equation)

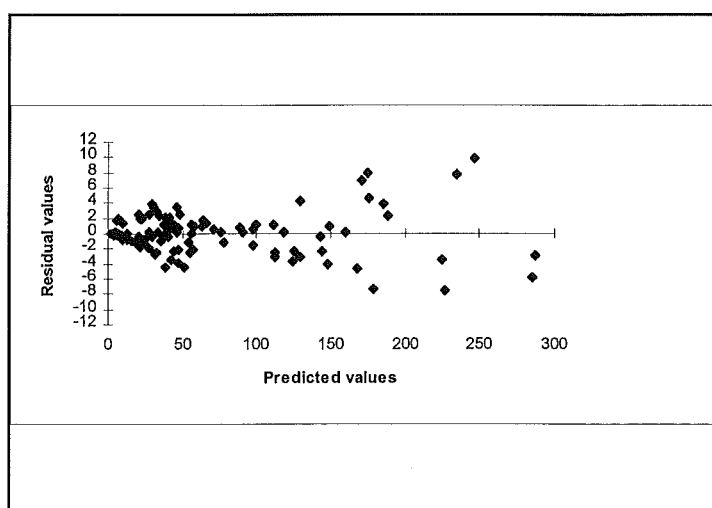
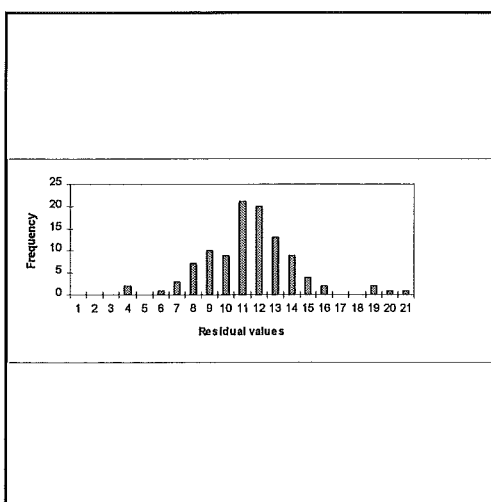


Fig 13. Residual and frequency plots for Volume (tree volume data from *E.regnans* equation)

Mean Top height and basal area vs Stocking

There are clear differences in the patterns of h_{100} and G growth for the three trials, as shown in Figs 14 and 15, a manifestation of the different variables influencing the growth of trees in each of the three locations. For trial cct32, however, lower stockings appear to produce the highest h_{100} development, whereas for G the stocking range 2095-2769 seems to be the most desirable one, as even higher ones appear to induce greater mortality. In all cases there is a definite influence of stocking on the two variables h_{100} and G. The use of Nelder data brings into question the validity of ascribing the true impacts of different stocking levels

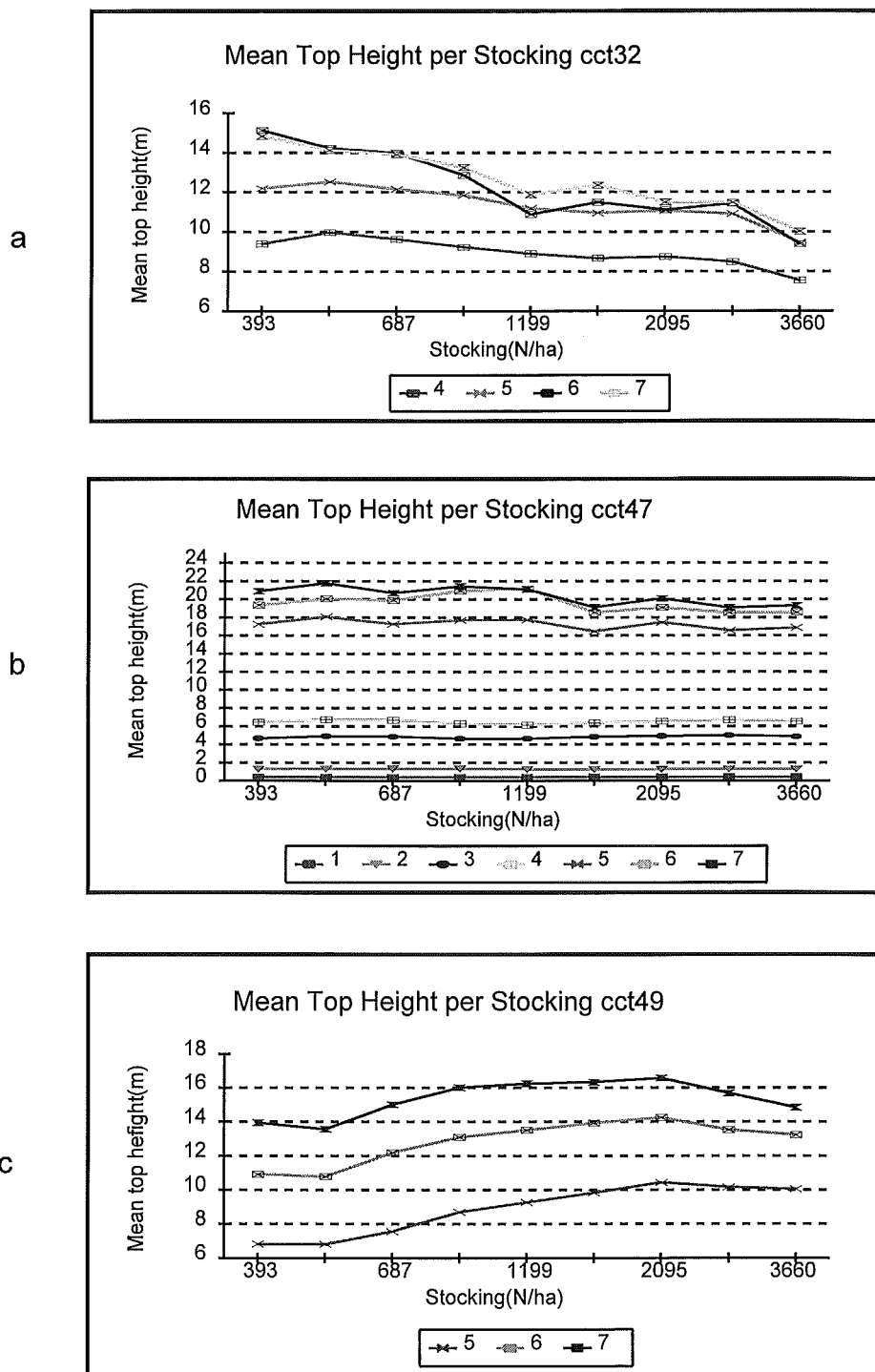


Fig 14. Mean Top Height per Stocking for three trials

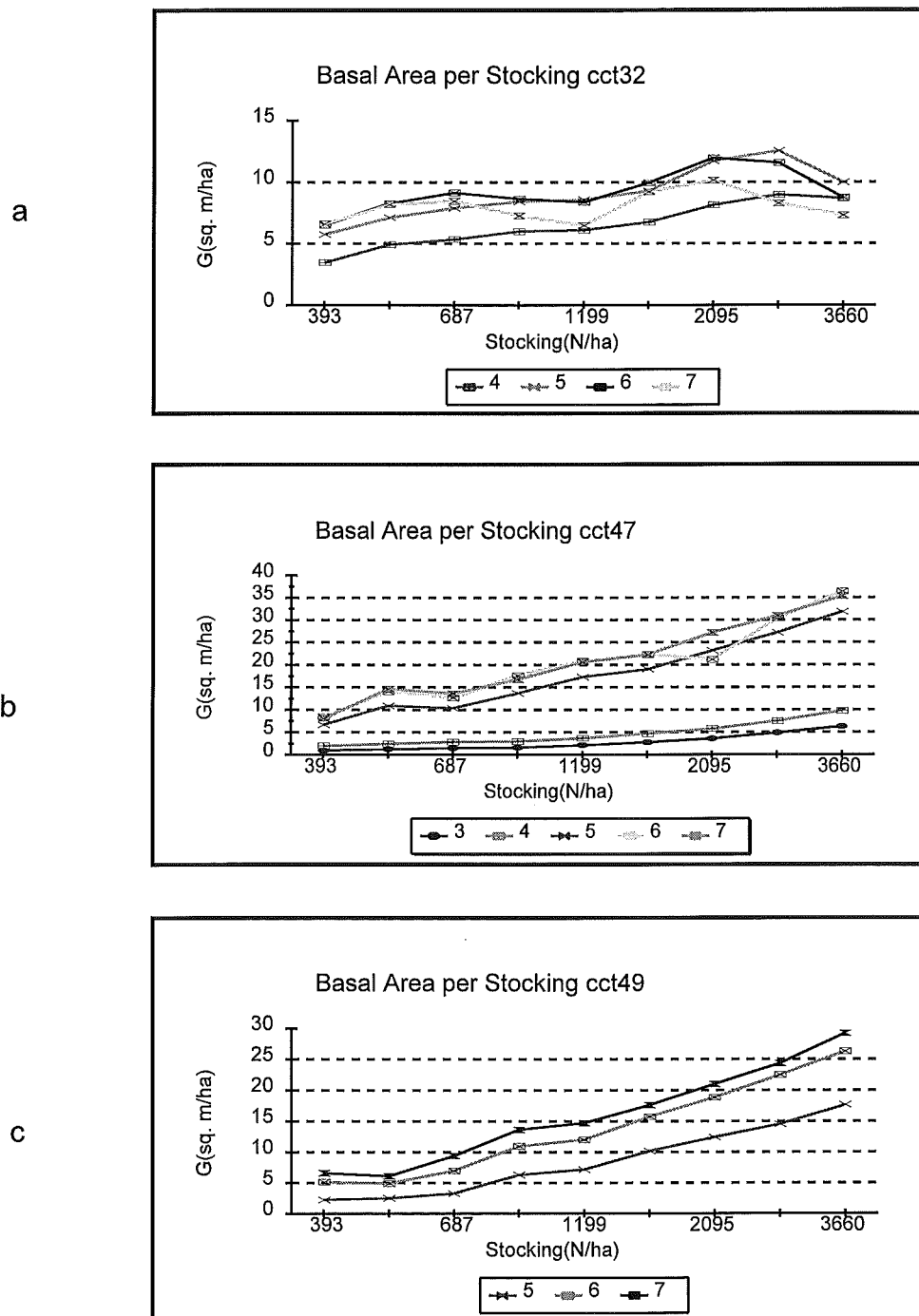


Fig 15. Basal Area per Stocking for three trials

Discussion

Basal area and mean top height modelling

The data used in this exercise are not of sufficiently high quality, or in preferred form because of the controversies surrounding the Nelder design. The losses of stems and the inaccuracies and inconsistencies of measurements are not unusual however, in long term experiments. Some of these problems and possible solutions have been discussed in the literature review and data justification sections of this report.

There is also no consensus among researchers about the use of least-squares regression techniques to estimate parameters using repeated measures. Some of literature shows that the correlation of errors in remeasurement data can be ignored under some circumstances (Woollons and Hayward, 1985, Sullivan and Clutter, 1972), but alternative methods of estimating regression parameters have been suggested, such as maximum-likelihood estimates (Sullivan and Clutter, 1972) and generalised least-squares (Ferguson and Leech, 1978; Davis and West, 1981). These alternative techniques have largely been proposed but rarely implemented as the gains are of limited practical value. West *et al.*, 1984 give an extended review of this problem. Remeasured data have in fact been used to estimate parameters over the years with few problems of bias. The estimates of coefficients are generally unbiased but the variances of the estimates tend to be larger than would be the case with independent data (Sullivan and Clutter, 1972). Statistical hypotheses about the regression can also not be strictly tested. Statistical hypotheses are one among the various tests used to assess the goodness of fit of the functions but

there are other ways of testing the fitting, through residual plots, mean residual, MSE, and other variables associated with these. Therefore remeasured data can be used to estimate regression parameters, if the lack of a true measure of variance is not critical and hypothesis tests are not contemplated. In this exercise least-squares regression was used on data with correlated errors and so standard hypothesis testing with F and t statistics were not used.

Log Reciprocal and Hossfeld functions for h_{100}

The SEE bounds of the log reciprocal were wider than for the Hossfeld function, in both overlapping and non overlapping data sets, Tables 3, 4, 5 and 6. The log reciprocal SEE bounds are slightly wider for non overlapping data than overlapping data and this is a pattern confirmed by the other testing indicators of skewness, kurtosis, mean residual, and residual plots. This points out to the importance of using a combination of these tests to segregate functions because some of the tests are insufficiently reliable on their own. This is true in the case of residual plots and histograms the shape of which can vary according to the chosen scale and class intervals respectively. The Hossfeld fit was acceptable for both overlapping and non overlapping data but the the fit to overlapping data showed some improvements over the fit to non overlapping ones. In view of these findings the Hossfeld was chosen as the appropriate projection function for mean top height.

Log Reciprocal and Hossfeld for G

The log reciprocal fits both overlapping and non overlapping basal area data better than the h_{100} data as has been shown in Tables 3 4, 5 and 6 and Figs 8 and 9. The

SEE parameter estimate bounds for the log reciprocal and Hossfeld functions for G are not distinctly different compared to the SEE of the same functions in the h_{100} fittings. The Hossfeld was the preferred choice in this case, because, on the basis of kurtosis, skewness, mean residual and residual plots, it appeared to be superior to the log reciprocal. The improvements resulting from the use of the Hossfeld function appear small but such small differences convert to large volume/ha differences when the G and h_{100} are input into the stand volume function.

Overlapping data were shown to be superior to non overlapping data especially in predicting height data, which is not what Borders *et al.*, 1987 found. Using what they called Souter's and Piennar's models, they found that each of the models behaved differently with overlapping data, non overlapping data or longest interval data. Souter's model did well with non overlapping interval data and Piennar's model did well with longest interval data. Longest interval was not investigated in this exercise. This implies that the suitability of data intervals is dependent on the model being applied to the data.

Non linear least-squares models were chosen to be applied, because they allow characteristics of the sigmoid curve, with a slow early increment, linear accelerated increment, inflection point, and upper asymptote to be expressed. Cumulative growth of most biological organisms assume the sigmoid curve (Avery, 1967; Carron, 1968 and Spurr, 1952) and fitting non linear functions is more representative to this biological phenomena than linear, multilinear or curvilinear functions.

Non linear models, however, present one challenge, namely the estimation of parameters of the equations through partial differentiation which is not as straightforward as in linear and multilinear functions. All the methods used in SAS,

Gauss-Newton, Marquardt, Gradient and DUD require initial parameter estimates as a starting point. The first three methods also require partial derivatives to be declared but DUD, which was used here, does not. This normally works very well but one has to be well acquainted with the responses of the model under different sets of parameter combinations. Experience is the key to good initial parameter estimates. It is also important to know which variables to include and how to combine them in the model. In this case when initial stocking was included in the differential functions, there was a very small improvement in the ESS, MSE, mean residual, SEE, skewness, kurtosis and residual plots. This improvement appeared small until the projected G and h_{100} were converted to volume/ha by the stand volume equation. A worked example to show the differences between future G and h_{100} estimated from differential functions with and without the initial stocking and the resultant volume/ha is shown in Appendix C for the Hossfeld function.

Basal area and mean top height vs stocking.

The trials used here were set up to help determine desirable stocking levels and it has been quite clear that G and h_{100} plotted against stocking do not on their own reach satisfactory enough conclusions. Initial spacing affects stand G growth rate differently from G growth of individual trees (Evert, 1971). The largest basal area per tree would be expected to be in the widest spacings and the largest basal area/ha would on the contrary be expected to be in the closest spacings. Therefore one would need to focus on the tree and not just on the stand variables.

There was a positive relationship between basal area and stocking but there was no clear pattern for height. This observation agrees with some documented

evidence, which shows that G tends to be influenced by spacing more than height. There are, however, conflicting views about height: it has been reported to be both responsive and unresponsive to changes in stocking (Evert, 1971, Lanner, 1985, Omiyale, 1990, Hamilton and Christie 1974, Liegel *et al* 1985, Van Laar and Bredenkamp 1979 and Turnbull *et al.*, 1993). Schonau and Coetzee, 1989 give a comprehensive review of the relationship between height and stocking and it is clear that mean height and not mean top height or dominant height can respond to spacing changes. The literature they quoted goes on to indicate that the response of mean height, mean top height or dominant height to spacing/stocking is also influenced by age, species and site quality, but generally mean top height is not affected by spacing as much as diameter and basal area are. Lanner, 1985 and Avery, 1967, investigated the relative independence of height from stocking and they give physiological explanations to these observations. Lanner, 1985 found out that for the family Pinaceae the axial growth of buds depend on the conditions of the previous year while cambial growth reflects current conditions, which means shoot growth is more predetermined than cambial growth. Although this explanation was not for *Eucalyptus*, it is a theory that could be pursued because the response to spacing of Pinaceae is the same for Eucalypts.

The analysis here of stocking as it affects G and h_{100} has merely scratched the surface of the subject and the variability of views on h_{100} are reflected in Figs 12 (a), (b) and (c). Such a small data base is too small to draw definite conclusions. The continuous decrease of h_{100} in Fig 12 (a) is most likely caused by high mortality.

Conclusions and Recommendations

This study of h_{100} and G growth of *Eucalyptus* in Zimbabwe has reinforced conclusions which are consistent with those that have been made previously about other regional growth and yield models, and drawn others that are unique to the set of data used here.

1. Design of data sources, data collection procedures, and management and storage of data have all been found to be very important aspects of growth and yield modelling. Data set discrepancies in G and h_{100} growth trends for trial cct47 emphasised this claim; a big difference in growth between ages 4 and 5 made it impossible to tell the exact growth path. It is imperative to remedy such errors right from the beginning because users of growth data may not necessarily be those involved in the design, collection and management procedures in the first place, considering the amount of time it takes to collect enough data to build a model. This is particularly relevant to Zimbabwe, where high turnover in research staff is not uncommon.

2. Some authors have reservations about Nelder trials but in a situation where psp data are not available, Nelder trial data can be useful for modelling growth as is shown in this study.

3. Modelling growth with non-linear functions proved superior to multi-linear functions because of the non-linear nature of tree and stand growth over time. Considerable

experience and good understanding of the behaviour of the functions in relation to various parameter and predictor variable combinations is the key to good initial parameter estimation.

4. In fitting the differential growth and also stand volume functions tried here, graphical plots of residuals, normality plots, skewness, kurtosis, ESS, and MSE proved to be very powerful tools for checking the goodness of fit of functions.

i. Both functional forms used to characterise G and h_{100} , namely the Hossfeld and log reciprocal, performed better with overlapping than with non overlapping data.

ii. For h_{100} the Hossfeld function was superior to the log reciprocal function.

iii. For G the Hossfeld function was the preferred choice but it is only marginally superior to the log reciprocal function.

iv. Inclusion of initial stocking improves the fit of the differential functions slightly as shown by the indicators SSE, ESS, MSE, graphical plots of residuals, skewness and kurtosis. These small changes convert to large differences in volume yields.

v. A linear stand volume equation based on G/ha and h_{100} , was clearly superior to a stand volume function based on age, stocking, h_{100} and G .

5. Tree volume functions for *E.grandis* were not available from Zimbabwe and so New

Zealand functions were used instead to show their role in developing growth models. Relevant functions for Zimbabwe will need to be substituted as soon as is practicable.

6. The growth trends of G/ha and h_{100} over time are similar for each stocking as well as for locations in which each of the 3 trials were found. This allowed the data to be combined.

7. In all the three trials, G growth is positively related to stocking but there is no such clear pattern for h_{100} . In cct 32, the height trends are the same as for G , in trial cct49 the relationship is negative and in trial cct47 there is very little difference over the 9 stocking levels.

The projection functions and volume equations developed in this exercise are not refined enough to be used for industrial purposes but the modelling methodology can be used to guide future *Eucalyptus* modelling, especially when some of the inconsistencies in the data are resolved and validations are carried out. The limitations of the available data and unavailability of typical *Eucalyptus* yields from the particular areas in Zimbabwe meant that validations of the models could not be carried out.

For data that come from separate environments like the one used here it would make a big difference to include environmental and other site variables in the model. The use of dummy variables would also improve the capacities of the models when larger and wider data set is available.

Summary

Three trials, all under the age of 7 and in which measurements had been taken from ages 1 or 3, were used to model stand height and basal area growth of *E. grandis* in Zimbabwe. Two of the trials, cct47 and cct49 are located in the eastern part of the country, at Gungunyana (rainfall: 1410 mm/annum, altitude: 1050 m above sea level) and Muguzo (rainfall: 1410 mm/annum, altitude: 1483 m above sea level) respectively. The third, cct32 is located further inland, at Mtao (rainfall: 755.5 mm/annum, altitude: 1477 m above sea level). Growth trends justified combining cct47 and cct49, while cct32 was included to increase the data base and also because it is geographically and climatically close to the other two trials.

The spreadsheet package, Quattro pro version 5.00 was used to calculate G , h_{100} and to arrange and edit the data before they were entered into SAS for statistical evaluation, further editing and model fitting. Scatter plots in the spreadsheet and graphical residual plots in SAS were used to remove outliers. Most of the outliers in the residual plots were traced back to the original data set. In fitting functions data were arranged either as overlapping or non overlapping sets and the latter was found to provide superior fits.

Two New Zealand tree volume functions were used, for expediency, because none was available from Zimbabwe. They can be easily replaced when Zimbabwean functions become available. A functional non-linear stand volume equation (one based on G and h_{100}), was found to be superior to a direct multi-linear stand volume function based on age, stocking, G and h_{100} . In order to take advantage of this, differential G and h_{100} growth functions over time were developed. Two types, the Hossfeld and log reciprocal functions were fitted by means of non-linear regression procedures in SAS.

The parameters for the functions were altered successively to improve the fit. For h_{100} , the residual plots, MSE, ESS, skewness, kurtosis and mean residual all favoured the Hossfeld function. The Hossfeld and log reciprocal for G did not show any great differences in goodness of fit, but overall the Hossfeld gave a better fit. Although not explored here, further refinements can be achieved by assigning dummy variables to account for variation due to locality. An approach like this, which takes into account the local variation of data in one model, is better than having separate localised models because of its more efficient use of a limited data base.

A short evaluation of the trends of G and h_{100} plotted over stocking was done to compare growth responses with available documented research information. Comparisons depend on where the trial is located and what variable is under consideration. For example basal area increases from lower to higher stockings for all three trials but not quite the same pattern emerges for h_{100} ; the trend is the same in cct32, but is the reverse in cct49 and there is little difference in h_{100} growth over the 9 stocking levels in cct47.

Acknowledgements

I wish to thank my supervisor, Dr. A.G.D. Whyte for being so patient with me over the whole time that I have been in the School of Forestry. I am also grateful to Forestry Commission who provided me with the data to do my report. I will also not forget the assistance I got from the staff of the School and fellow postgraduate students. Last, but most important, I would like to thank the New Zealand government through the Ministry of Foreign Affairs and Trade for providing this scholarship.

References

Avery, T.E. and Burkhardt, H.E. 1983. Forest Measurements. 3rd edition. McGraw-Hill Series in Forest Resources. New York.

Barret, R.L. and Mullin, J.L. 1968. A Review of Introductions of Forest Trees in Rhodesia. *The Rhodesia Bulletin of Forestry Research No. 1*.

Borders, B.E., Bailey, R.L. and Clutter, M.L. 1987. Forest Growth Models: Parameter estimation using real growth series. *In* Ek, A.R., Shifley, S.R. and Burk, E.T. (eds), Forest Growth and Yield Modelling and Prediction. Proceedings of the IUFRO Conference, Minneapolis, Minnesota, Aug 23-27. *USDA Forest Service, North Central Expt. Station. Gen. Techn. Report. NC120, vol 2:660-667*.

Bredenkamp, B.V. 1984. The CCT concept in Spacing Research-Review. *In* Grey, D.C., Schonau, A.P.G. and Schutz, C.J. (eds), Proceedings of the IUFRO Symposium on Site and Productivity of Fast Growing Plantations, Pretoria and Pietermaritzburg, 30 April - 11 May 1984. *South Africa Forestry Research Institute, Pretoria, Vol. 1:313-332*.

Bruce, D. and Wensel, C. 1987. Modelling Forest Growth: Approaches, Definitions and Problems. *In* Ek, A.R., Shifley, S.R. and Burk, T.E. (eds), Forest Growth Modelling and Production. Proceedings of the IUFRO conference, Minneapolis, Minnesota, August 23-27. *USDA Forest Service, North Central Expt. Station. Gen. Techn. Report NC 120, vol 1:1-6*.

Burgers, T.F. 1971. The Correlated Curve Trend (CCT) Method used for Growth Prediction in Thinned Stands of *Pinus patula*. *Forestry in South Africa* 12:11-17.

Burley, J., Armitage, F.B., Barnes, R.D., Gibson, G.L., Hardcastle, P.D., Huguet, L., Plumptre, R.A. and Wood, P.J. 1989. Forestry Research in Eastern and Southern Africa. *Tropical Forestry papers No. 19, Annex 22. Oxford Forestry Institute*.

Carron, L.T. 1968. An Outline of Forestry Mensuration with special reference to Australia. *Austr. Natl. Univ. Press. Canberra.* 244pp.

Clutter, J.L., Fortson, J.C., Piennar, L.V., Brister, G.H. and Bailey, R.L. 1983. Timber Management: A Quantitative Approach. John Wiley and Sons, USA. 333 pp.

Clutter, J.L. 1963. Compatible Growth and Yield Models for Loblolly Pine. *For. Sci.* 9(3):354 - 371.

Crockford, K. 1993. pers. comm.

Davis, A.W. and West, P.W. 1981. Remarks on "Generalised least Squares Estimation of Yield functions", by I. S. Ferguson and J. W. Leech. *For. Sci.* 27(2):233-239.

Draper, N.R. and Smith, H. 1981. Applied Regression Analysis. 2nd edition. John Wiley and Sons, New York, 709pp.

Evert, F. 1971. Spacing Studies: A Review. *Can. For. Ser. Dept of the environ. Forest Management Inst. Informal Report. FMR-X-37.*

FAO, 1985. The Ecological Effects of *Eucalyptus*. *FAO Forestry Paper, 59. Rome.* 87 pp.

Ferguson, I.S. and Leech, J.W. 1978. Generalised least Squares Estimation of Yield Functions. *For. Sci.* 24(1):27-42.

Forestry Commission (Zimbabwe), 1993. Progress Report 1989-92. 14th Commonwealth Forestry Conference, 13-18 September 1993, Kuala Lumpur, Malaysia. 6pp.

Forestry Commission (Zimbabwe) Survey. 1989-90. A Survey of Plantations and Roundwood Processing in Zimbabwe. 67 pp.

Gujarati, D. 1970. Use of Dummy Variables in Testing for Equality Between Sets of Coefficients in Linear Regressions: A Generalization. *The American Statistician* 25(4): 19-22.

Hamilton, G. H and Christie, J. M, 1974. Influence of Spacing on Crop Characteristics and Yield. *Forestry Commission (UK) Bulletin No. 52*.

Kanyemba, I. 1984. Contribution of Forestry and Forestry Industries to the Economy of Zimbabwe. Forestry dissertation, University of Canterbury, Christchurch, New Zealand.

Lanner, R.M. 1985. On the Insensitivity of Height Growth to Spacing. *For. Ecol. and Mgmt.* 13:143-148.

Liegel, L.H., Balmer, W.E. and Ryan, G.W., 1985. Honduras Pine Spacing Trial Results in Puerto Rico. *South. Jour. Appl. Sci.* 9(1):70-75.

Liu Xu, 1990. Growth and Yield of Douglas fir Plantations in the Central North Island of New Zealand. Phd. thesis, University of Canterbury, Christchurch, New Zealand. 233 pp.

Moser, J.W. Jr. and Hall, O.F 1989. Deriving Growth and Yield Functions for Uneven Aged Forest Stands. *For. Sci.* 15(2):183 - 188.

Nelder, J. A 1962. New Kinds of Systematic Designs for Spacing Experiments. *Biometrics* 18:283-307.

Omiyale, O. 1990. Growth of *Pinus Caribaea* under four Spacings in the Savannah Zone of Nigeria. In James, R.N. and Tarlton, G.L. (eds)., New Approaches to Spacing and Thinning in Plantation Forestry. Proceedings of a IUFRO Symposium held at the Forestry Research Institute, Rotorua, New Zealand, 10-14 April 1989. pp 223-228.

Ralston, M.L. and Meek, C.L. 1979. Encyclopedia of Computer Science. 1st edition. Petrocelli-charter, New York.

SAS Institute Inc., 1990. Users's Guide. Version 6, vol 1 and 2.

_____ SAS Language. Reference Version 6.

_____ SAS Procedures Guide Version 6.

Schonau, A.P.G. and Coetzee, J. 1989. Initial spacing, Stand Density and Thinning in Ecalyptus Plantations. *For. Ecol. and Mgmt.* 29:245 - 266.

Spurr, S. H. 1952. Forest Inventory. The Ronald Press Co. New York. 476pp.

Sullivan, A.D. and Reynolds, R.M. Jr. 1976. Regression Problems from Repeated Measurements. *For. Sci.* 22(4):382 - 385.

Sullivan, A.D. and Clutter, J.L. 1972. A Simultaneous Growth and Yield Model for Loblolly Pine. *For. Sci.* 22:382-385.

Temu, J. M. 1992. Forecasting Yield of Douglas fir in the South Island of New Zealand. Phd. thesis, University of Canterbury, Christchurch, New Zealand.

Turnbull, C.R.A., Macleod, D.E., Ratkosky, C.L., Mummery, D.C. and Bird, T. 1993. Comparative Early Growth of Eucalypts species of the Subgenera Monocalyptus and Symphyomyrtus in Intensively managed Plantations in Southern Tasmania. *Austr. For.* 56(3):276-286.

Van Laar, A. and Bredenkemp, B. 1979. The Effect of Initial Spacing on some Growth Parameters of *Eucalyptus grandis*. *South African For. Jour.* 111:58-63.

Villanueva, T.R. 1992. Intergrated Yield Forecasting and Harvest Scheduling in a Tropical Pine Plantation in Fiji. Phd. thesis, University of Canterbury, Christchurch, New Zealand.

West, P.W., Ratkowsky, D.A. and Davis, A.W. 1984. Problems of Hypothesis testing of Regression with Multiple Measurements from individual sampling units. *For. Ecol. and Mgmt* 7:207-224.

Woollons, R.C., Whyte, A.G.D. and Liu Xu. 1990. The Hossfeld Function: An alternative model for depicting stand growth and yield. *Jour. Jap. Assoc. For. Stat.* No. 15:25-35.

Woollons, R.C. and Hayward, W.J. 1985. Revision of a Growth and Yield Model for Radiata Pine in New Zealand. *For. Ecol. and Mgmt* 11:191 - 202.

Appendix

Appendix A.

Location of the Eastern Highlands of Zimbabwe and sites of the trials used in this study.

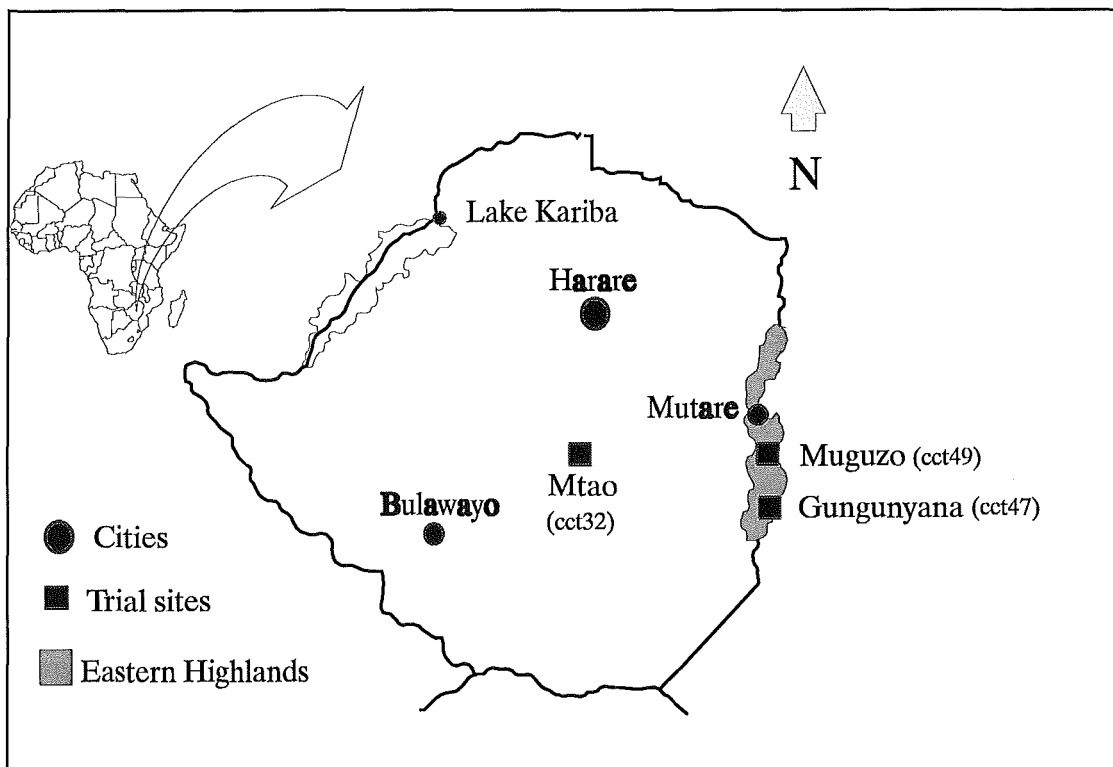


Fig 16. Map of Zimbabwe showing the location of Eastern Highlands and Trial sites
(not drawn to scale)

Appendix B1.

This section contains tables with the results of running models 1 to 8 (results section) without the stocking variable. Tables 9 and 10 are for non overlapping data and Tables 11 to 12 are for overlapping data.

Table 9 .Comparison of fitting indices for log reciprocal and Hossfeld functions fitted to non overlapping data.

| Equation | Para-meter | Estimate | SEE | ESS | MSE |
|----------------------------------|---------------------|----------------------------|--------------------------------|------------|----------|
| Log Reciprocal (h_{100}) (1) | α β | 3.678054230 0.726872628 | 0.18344899768 0.09867425915 | 158.052839 | 1.796055 |
| Hossfeld (h_{100}) (5) | α β | 0.042981250 2.132975447 | 0.00282364927 0.18311093893 | 155.143530 | 1.762995 |
| Log Reciprocal (G) (1) | α β | 4.002718759 0.632814846 | 0.39636408728 0.19888294886 | 366.610766 | 5.237297 |
| Hossfeld (G) (5) | α β | 0.024833669 1.431273619 | 0.00476682697 0.26565592283 | 371.701238 | 5.310018 |

Table 10. Comparison of fitting indices for log reciprocal and Hossfeld functions fitted to non overlapping data.

| Equation | Skewness | Kurtosis | Mean Residual | n |
|----------------------------------|----------|----------|---------------|----|
| Log Reciprocal (h_{100}) (1) | 0.532269 | -0.4687 | 1.330332 | 90 |
| Hossfeld (h_{100}) (5) | 0.313415 | -0.57083 | 0.071487 | 90 |
| Log Reciprocal (G) (1) | 0.477049 | 0.855548 | -0.4517 | 72 |
| Hossfeld (G) (5) | 0.28746 | 0.57852 | 0.015676 | 72 |

Table 11. Comparison of fitting indices for log reciprocal and Hossfeld functions fitted to overlapping data.

| Equation | Parameter | Estimate | SEE | ESS | MSE |
|----------------------------------|---------------------|----------------------------|--------------------------------|-------------|-----------|
| Log Reciprocal (h_{100}) (3) | α β | 3.750745387 1.040256250 | 0.15198005352 0.09718963037 | 4423.836891 | 16.506854 |
| Hossfeld (h_{100}) (7) | α β | 0.041830036 3.127240830 | 0.00186067494 0.10258589250 | 3342.778083 | 12.473053 |
| Log Reciprocal (G) (3) | α β | 3.510952237 1.844933972 | 0.14735204747 0.25943024588 | 6414.201397 | 37.953854 |
| Hossfeld (G) (7) | α β | 0.037052100 3.833911606 | 0.00306025085 0.29732108368 | 6788.662111 | 40.169598 |

Table 12. Comparison of fitting indices for log reciprocal and Hossfeld functions fitted to overlapping data.

| Equation | Skewness | Kurtosis | Mean Residual | n |
|------------------------------|----------|----------|---------------|-----|
| Log Reciprocal (h_{100}) | 0.2837 | -0.75626 | 0.251733 | 270 |
| Hossfeld (h_{100}) | 0.117841 | -0.40108 | -0.00876 | 270 |
| Log Reciprocal (G) | 0.693222 | 0.530932 | -0.41986 | 171 |
| Hossfeld (G) | 0.365892 | 0.037457 | -0.28948 | 171 |

Appendix B2.

The graphs in this section show the results of running models 1 to 8 without the initial stocking variable.

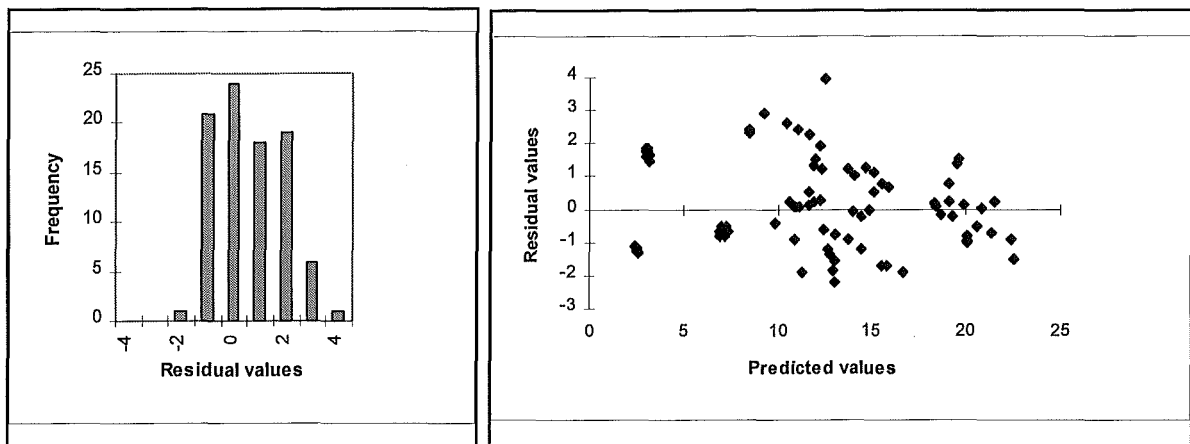


Fig 17. Residual and frequency plots for height, log reciprocal fitted to non overlapping data

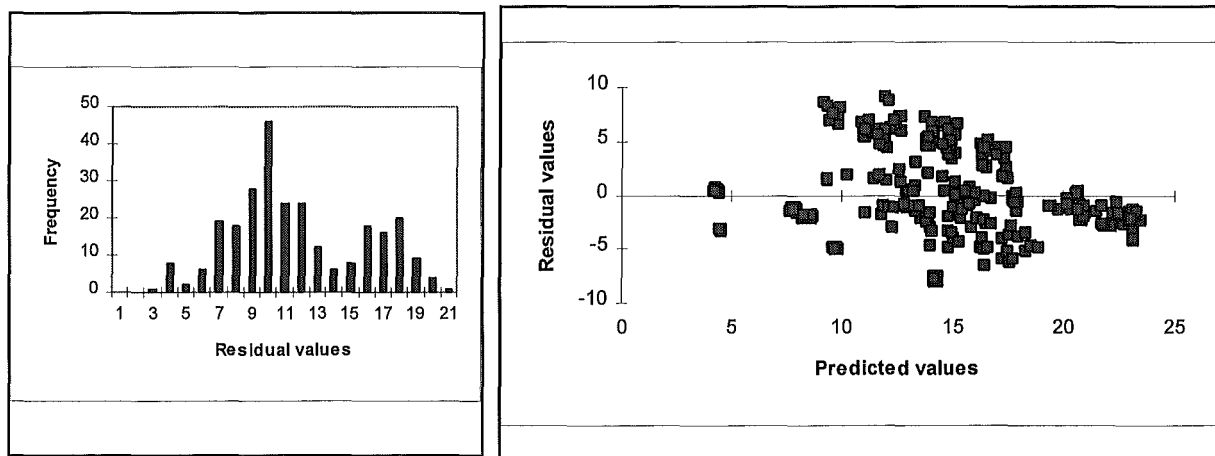


Fig 18. Residual and frequency for height, log reciprocal function fitted to overlapping data.

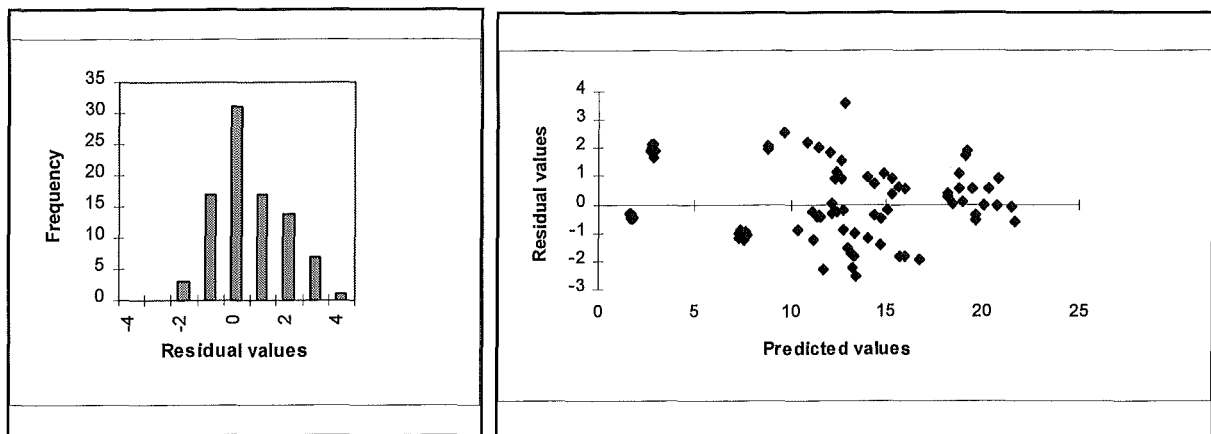


Fig 19. Residual and frequency plots for height, Hossfeld function fitted to non overlapping data.

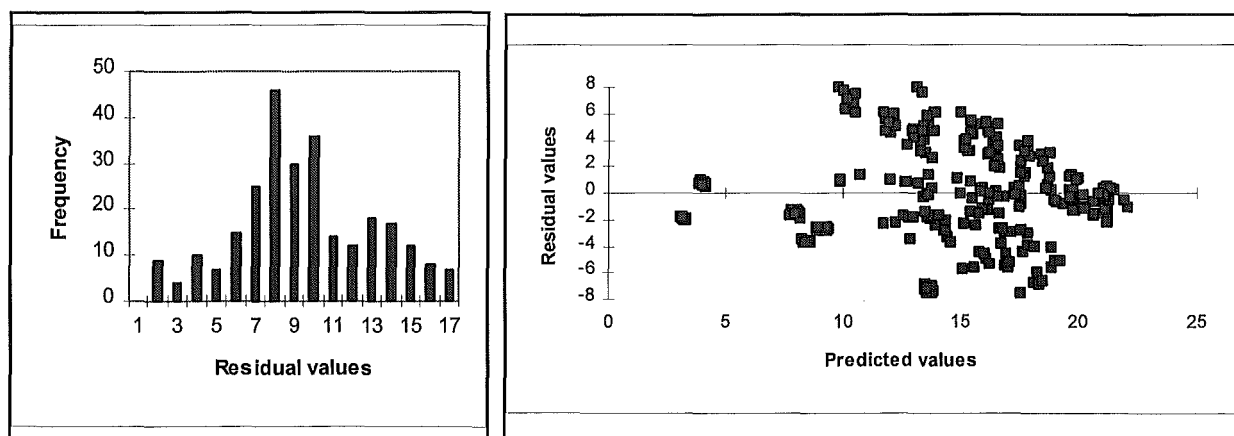


Fig 20. Residual and frequency plots for height, Hossfeld function fitted to overlapping data.

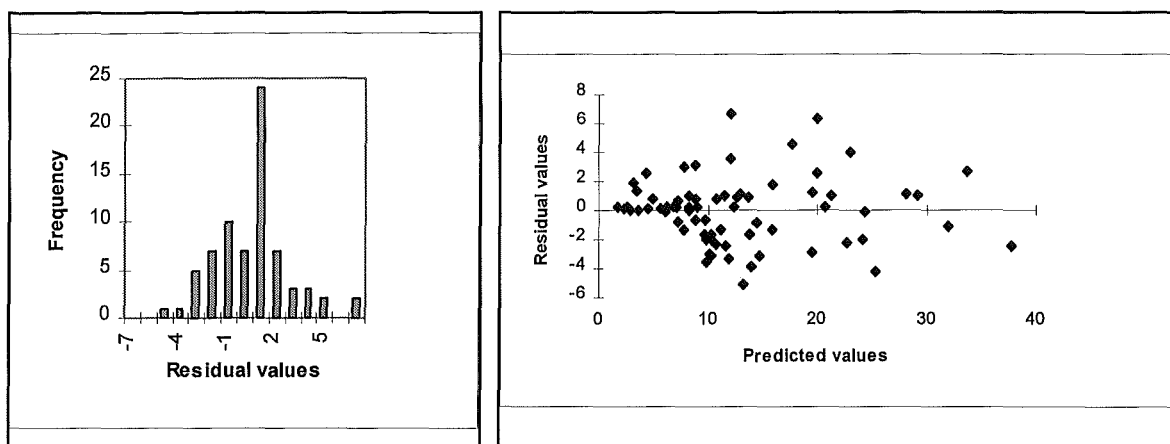


Fig 21. Residual and frequency plots for basal area, log reciprocal function fitted to non overlapping data.

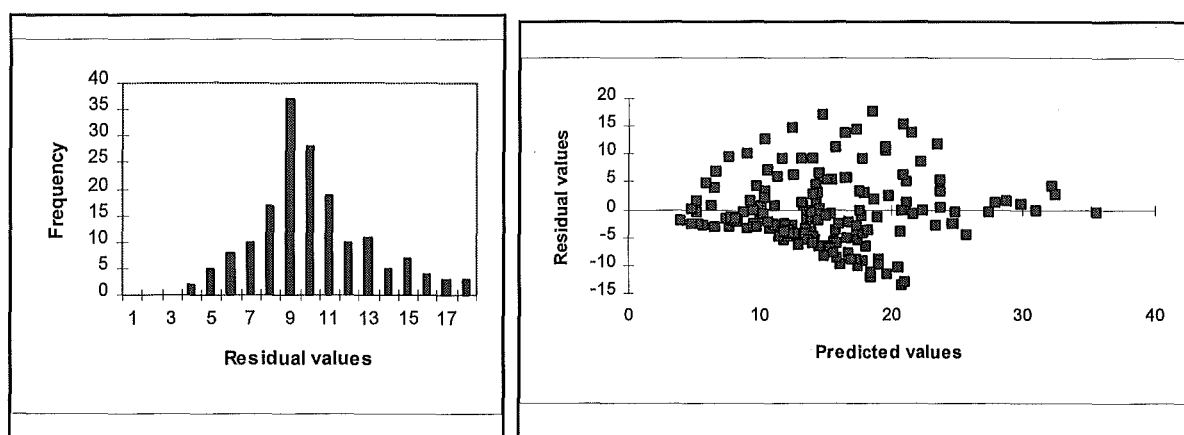


Fig 22. Residual and frequency plots for basal area, log reciprocal function fitted to overlapping data.

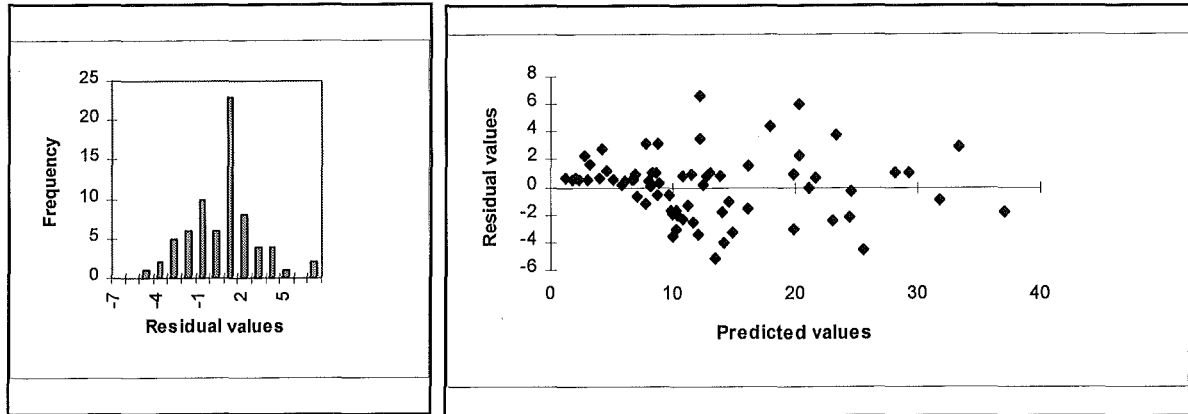


Fig 23. Residual and frequency plots for basal area, Hossfeld function fitted to non overlapping data.

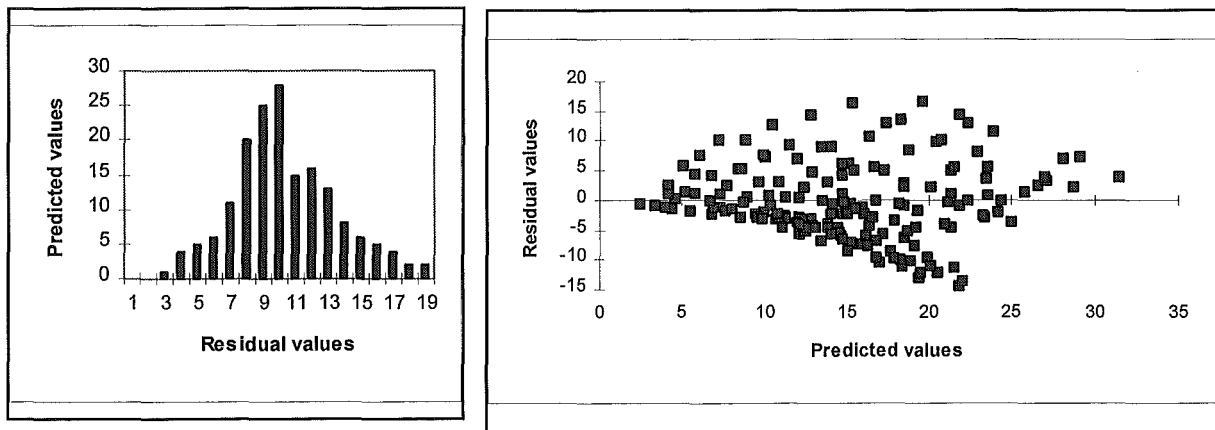


Fig 24. Residual and frequency plots for basal area, Hossfeld function fitted to overlapping data (no initial stocking variable).

Appendix C

The projection equations were applied to the same data set from which the parameters were estimated, to show the effect on volume yields, of the initial stocking variable. In the extract of the spreadsheet below, Table 13, basal area and height are highlighted for age 4 and 7. The G and h_{100} at age 4 will be used in the two projection equations, log reciprocal and Hossfeld to show that the small differences in residual plots, MSE, SSE and the other tests will translate to quite significant volume/ha estimates.

Table 13. Extract of the form in which the data was modelled.

| Initial Stocking | T1 | T2 | h_{100} at T1 | h_{100} at T2 | G at T1 | G at T2 |
|------------------|----|----|-----------------|-----------------|---------|--------------|
| 3660 | 4 | 5 | 7.755 | 9.462 | 8.866 | 9.977 |
| 3660 | 5 | 6 | 9.462 | 9.378 | 9.977 | 8.724 |
| 3660 | 6 | 7 | 9.378 | 10.007 | 8.724 | 7.28 |
| 3660 | 1 | 2 | 0.42311 | 1.3 | | |
| 3660 | 2 | 3 | 1.3 | 4.8511 | | 6.18 |
| 3660 | 3 | 4 | 4.8511 | 6.489 | 6.18 | 9.688 |
| 3660 | 4 | 5 | 6.489 | 16.842 | 9.688 | 31.73 |
| 3660 | 5 | 6 | 16.842 | 18.542 | 31.73 | 36.328 |
| 3660 | 6 | 7 | 18.542 | 19.3 | 36.328 | 35.28 |
| 3660 | 5 | 6 | 10.038 | 13.224 | 17.62 | 26.272 |
| 3660 | 6 | 7 | 13.224 | 14.836 | 26.272 | 29.219 |

Log reciprocal without stocking

$$Y_2 = Y_1^{(T1/T2)^\beta} \times \exp(\alpha(1 - (T1/T2)^\beta))$$

Log reciprocal with stocking

$$Y_2 = Y_1^{(T1/T2)^{\beta+\gamma \times N}} \times \exp(\alpha(1 - (T1/T2)^{\beta+\gamma \times N}))$$

Hossfeld without stocking

$$Y_2 = 1 / ((T1/T2)^\beta \times 1/Y_1 + (\alpha) \times (1 - (T1/T2)^\beta))$$

Hossfeld with stocking

$$Y_2 = 1 / ((T1/T2)^{\beta+\gamma \times N} \times 1/Y_1 + (\alpha) \times (1 - (T1/T2)^{\beta+\gamma \times N}))$$

Table 14. h_{100} and G estimates from Hossfeld functions, with and without the stocking variable

| | h_{100}/G at T2, initial stocking not included in equation | h_{100}/G at T2, initial stocking included in equation |
|------------------------|--|--|
| Hossfeld for h_{100} | 18.74837 | 17.66728 |
| Hossfeld for G | 21.00734 | 25.47534 |

Table 15. Prediction of volume at age 7, from G and h_{100} at age 4.

| | Volume/ha, Hossfeld function without the stocking variable | Volume/ha, Hossfeld function with the stocking variable |
|---|--|---|
| Individual tree volume derived from all NZ eucalypts function | 163.4567 | 191.8541 |
| Individual tree volume derived from <i>E.regnans</i> function | 161.6388 | 187.5134 |